ELASTIC ANALYSIS OF SPATIAL SHEAR WALL SYSTEMS WITH FLEXIBLE BASES

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SUMMARY

The static linear elastic analysis of spatial shear wall systems with flexible base foundations subjected to lateral loading is considered in this investigation. The discrete force method is adopted, in which the base flexibility is modelled by vertical and rotational springs. The soil and structure were coupled by imposing compatibility conditions along the discrete force method interfaces. The proposed method enforces compatibility of deformation on the relative vertical displacement between adjacent release positions, which are established at assumed contraflexural points of the connecting beams which link the slab together and also at storey level intervals along the intersection lines of adjoining non-coplanar walls. The solution of the compatibility conditions provides the redundant shear forces at the assumed contraflexural positions and the stress resultant solution is completed by statical considerations. The displacement solution corresponding to the stress resultants is then evaluated using the linear elasticity assumption. The objective of the current work is to provide a theoretical treatment of foundation–structure interaction, suitable for implementation in the discrete force method. In addition, the work presents results obtained with the discrete force modelling of soil interaction, and discusses its applicability to some representative problems.

1. INTRODUCTION

The majority of methods of analysis commonly used by engineers for the design of shear wall structures make the assumption that the structure is built into a rigid foundation. Such a procedure simplifies the mathematical analysis of the problem, and may often be sufficiently accurate for all practical purposes. However, depending on the form of structure and the particular soil conditions encountered, it may be considered desirable to estimate the effects of differential settlement produced by foundation movement.

The first study of the effect of flexible bases was reported by Coull1, 2 who treated rotational and vertical base flexibility separately, employing the continuum approach, and then superimposed the results to assess the general case of coupled rotational and vertical base boundary conditions. In a subsequent study, Tso and Chan3 examined simultaneous vertical and rotational movement at the bases of the coupled walls. Pekau4 considered the importance of base flexibility on the nonlinear behaviour of coupled shear walls subjected to pseudo-static lateral loading using the classic continuum method.

So far, only a few papers have dealt with the analysis of three dimensional soil–structure interaction problems. Swaddiwudhipong et al.5 were the first to investigate three dimensional
flexible foundations using the continuous approach method. Spyrakos and Beskos were among the first authors to have investigated two and three dimensional flexible foundations modelled with finite elements and coupled to halfspaces represented by boundary elements. It is well known that shear walls are sensitive to the flexibility of the foundations, so that there is an obvious need to examine the importance of base flexibility for three dimensional structures under realistic soil conditions.

Four methods of analysis may be contemplated for the analysis of shear walls subjected to lateral loading: namely: (1) frame analogy; (2) finite element formulation; (3) continuum method; and (4) discrete force method. Of these, the discrete force approach is especially well suited and is therefore adopted in this study. The present paper shows how the elastic formulation established for coupled planar walls with flexible bases may be extended to three dimensional shear wall arrangements. A particular advantage, owing to the discrete nature of the force approach, is that it offers almost total flexibility in respect of wall plan arrangement and elevational irregularities.

2. DISCRETE FORCE METHOD

The discrete force approach to shear wall analysis has been presented previously for the linear elastic analysis of planar walls by Johnson and Choo and for spatial shear wall systems by

Figure 1. Example shear wall system: (a) view of plan, (b) part front elevation
Johnson and Nadjai. The method has also been extended to the elasto-plastic condition by Johnson and Nadjai, assuming that plasticity is restricted to the connecting beams. Further improvement of the method has been developed for the elastic and elasto-plastic analysis of planar coupled shear walls with flexible bases by Nadjai and Johnson. The method employs an assumption of contraflexural points at the mid-points of beams connecting co-planar walls and at the junctions of non-planar walls or non-planar beam/wall intersections (Figure 1). The system of contraflexural points established in this way divides the shear wall system into a number of individual wall components, which may have beam stubs at one or both sides (Figure 1(b)).

If the further assumption of rigid in-plane floor deformation is made, so that the lateral displacement profiles of each of the sub-divided components are similar, then, in the absence of interconnecting shears at the contraflexural points, the system is statically determinate. The axial load and bending moment responses to the applied lateral loading may therefore be readily established for all the wall components. Similarly, the axial loads and bending moments created by the interconnection shears may be established in terms of these unknown (redundant) quantities. The redundant shears may be determined by the enforcement of compatibility of displacement between vertically adjacent connection positions. Following the evaluation of the redundant shears, the displacement and stress resultant response of the complete system may be readily established.

3. COMPATIBILITY CONDITION

The set of redundant shear forces \( \{q\} \) may be determined from the enforcement of compatibility on the relative vertical displacements of adjacent released positions. Figure 2 shows a typical storey \( s \) at a general joint \( j \) which includes connecting beam stubs, although these may be eliminated by setting the appropriate stub length \( s \) and \( a \) to zero. For the beams \( s \) and \( s-1 \), which bound storey \( s \), the vertical separation of the cut position of beam \( s \) and that of beam \( s - 1 \) must be identical, regardless of whether this displacement is evaluated from the left hand (1) side or from the right hand (2) side. If the vertical distance between the cut points measured

![Figure 2. Joint connection at level storey connecting beam stubs](image)
from side 1 is \( h_{1y} \), then
\[
h_{1y} = h_i + \delta_{1a} - \delta_{1b} + \delta_{1c} + \delta_{1d} + \delta_{1e} \tag{1}
\]
In equation (1), the displacements \( \delta_{1a} - \delta_{1e} \) are due to: (a) wall axial extension; (b) wall bending deformation; (c) connecting beam bending deformation; (d) connecting beam shear deformation; (e) wall shear deformation. These displacements may all be expressed in terms of the redundant shears \( \{q\} \) and the applied loading, the appropriate expressions for which have been provided previously.  

By a similar argument in respect of side 2:
\[
h_{2y} = h_i + \delta_{2a} + \delta_{2b} - \delta_{2c} - \delta_{2d} - \delta_{2e} \tag{2}
\]
For deformation compatibility between the adjacent cuts:
\[
(\delta_{1a} - \delta_{2a}) + (\delta_{1c} + \delta_{2c}) + (\delta_{1d} + \delta_{2d}) + (\delta_{1e} + \delta_{2e}) = (\delta_{1b} + \delta_{2b}) \tag{3}
\]

4. FLEXIBLE BASES

The boundary condition follows the assumption of contraflexural points at the mid-points of beam connecting co-planar walls and at the junctions of non-planar walls or non-planar/wall intersections. Applied at the base, this involves consideration of the rotation of the walls and of relative base settlement, which are directly proportional to the bending moment and axial force, respectively, at the base of the wall. The stiffnesses of the flexible foundation are represented by translational and rotational springs with stiffness \( K_v \) and \( K_r \) at the base of each wall (see Figure 3). In the elasto-plastic range of behaviour, it is assumed that the walls and supporting soil remain elastic.

4.1. Elastic vertical movement

In this case, it is assumed that the walls remain vertical, but, owing to the elasticity of the foundations, a vertical displacement occurs which is directly proportional to the magnitude of the axial force at the base of the wall (see Figure 4(a)). The vertical displacements due to foundation movements for walls 1 and 2, \( \delta_{v1} \) and \( \delta_{v2} \), must therefore be taken into account when forming the compatibility equation at joint 1 for storey 1, and may be evaluated as
\[
\delta_{v1} = \frac{N_{11}}{K_{v1}}, \quad \delta_{v2} = \frac{N_{21}}{K_{v2}} \tag{4}
\]

4.2. Elastic rotational movement

In this case, it is assumed that the foundations rotate under the influence of the moments at the wall bases. As a result of the fundamental assumption that the walls rotate equally, the base level rotates as a rigid body, the components of base rotation \( \beta_x \), \( \beta_y \) will be proportional to the base moments and, for an elastic foundation, may be expressed as
\[
\beta_x = \frac{m_{e0}}{K_{\beta x}}, \quad \beta_y = \frac{m_{e0}}{K_{\beta y}} \tag{5}
\]
where \( m_{x0}, m_{y0} \) are the moments at the base in the \( x- \) and \( y- \) directions, respectively, and \( K_{gx}, K_{gy} \) are the rotational stiffnesses of the foundation system in the \( x- \) and \( y- \) directions.

The rotation of walls 1 and 2 (Figure 4(b)) are then given by

\[
\begin{bmatrix}
\beta_1 \\
\beta_2
\end{bmatrix} = \begin{bmatrix}
c_1 & s_1 \\
c_2 & s_2
\end{bmatrix} \begin{bmatrix}
\beta_x \\
\beta_y
\end{bmatrix}
\]

where \( c \) and \( s \) are the cosine and sine of the orientation of the walls.

Thus, from Figure 4(b), the additional movements due to base rotation are given by

\[
\delta_{\beta 1} = c_{1j1} \beta_1 \quad \text{and} \quad \delta_{\beta 2} = c_{2j2} \beta_2
\]

4.3. Base compatibility condition

If the movements due to base flexibility are taken into account, then equation (1) for the vertical distance between ground level and the first storey cut as measured from side 1 becomes

\[
h_{1z} = h_1 + \delta_{1a} - \delta_{1b} + \delta_{1c} + \delta_{1d} + \delta_{1e} + \delta_{v1} - \delta_{\beta 1}
\]
By a similar argument in respect of side 2:

\[ h_{2a} = h_i + \delta_{2a} + \delta_{2b} - \delta_{2c} - \delta_{2d} - \delta_{2e} + \delta_{v2} + \delta_{b2} \]  

(9)

For deformation compatibility between the adjacent cuts:

\[(\delta_{1a} - \delta_{2a}) + (\delta_{1c} + \delta_{2c}) + (\delta_{1d} + \delta_{2d}) + (\delta_{1e} + \delta_{2e})
+ (\delta_{v1} - \delta_{v2}) + (\delta_{b1} + \delta_{b2}) = (\delta_{1b} + \delta_{2b}) \]

(10)

Equation (10) is therefore used in place of equation (3) when forming the compatibility expression for storey 1 under a flexible base condition.
4.4. Stress resultant solution

By expressing a complete set of compatibility conditions in terms of the redundant shears and the applied loading, a set of flexibility equations is obtained:

\[ [F] \{q\} = \{r\} \] (11)

The vector \( \{r\} \) in equation (11) is dependent on the applied lateral loading, which can be evaluated analytically or numerically, and the equations may be solved to obtain the redundant shears as

\[ \{q\} = [F]^{-1}\{r\} \] (12)

The redundant shears may then be used to obtain the stress resultant response and, subsequently, the displacement profile, for the complete wall systems.8

4.5. Lateral displacement

The procedure for determining the local wall displacement \( \hat{\lambda} \) at any storey has been discussed previously.8 From the local wall displacements, the floor displacements in the overall axes may be obtained by a transformation to the global axes. The wall element shown in Figure 5, for example, has local axes \( \{s, n, \theta\} \) which are related to an overall set of axes \( \{x, y, \theta\} \) by the orientation angle of the element \( \alpha \) and the coordinates of the element’s centroid in the overall axes \( (\bar{x}, \bar{y}) \). By the assumed rigid in-plane behaviour of the storey floors, it follows that the longitudinal displacement of the wall element \( \hat{\lambda} \) may be related to the displacements of the complete wall system in the overall axes \( \{u, v, \theta\} \) by the rigid-body transformation

\[ \hat{\lambda} = cu + sv + (s\bar{x} - c\bar{y})\theta \quad \text{where} \quad s = \sin(\alpha) \quad \text{and} \quad c = \cos(\alpha) \] (13)

or

\[ \hat{\lambda} = \{t\}^t\{\Delta\} \] (14)

where \( \{t\} = \{c, s, r\}, \quad \{\Delta\} = \{u, v, \theta\} \) and \( r = s\bar{x} - c\bar{y} \).

Figure 5. Typical wall element
Applying equation (14) to any three non-parallel walls gives

\[
\begin{align*}
\lambda_1 &= \begin{bmatrix} c_1 & s_1 & (s_1 x_1 - c_1 y_1) \end{bmatrix} \begin{bmatrix} u \\ v \\ \theta \end{bmatrix} \\
\lambda_2 &= \begin{bmatrix} c_2 & s_2 & (s_2 x_2 - c_2 y_2) \end{bmatrix} \begin{bmatrix} u \\ v \\ \theta \end{bmatrix} \\
\lambda_3 &= \begin{bmatrix} c_3 & s_3 & (s_3 x_3 - c_3 y_3) \end{bmatrix} \begin{bmatrix} u \\ v \\ \theta \end{bmatrix}
\end{align*}
\] (15)

hence

\[
\{\lambda\} = [T]\{\Delta\} \quad \text{or} \quad \{\Delta\} = [T]^{-1}\{\lambda\}
\] (16)

where \([T]\) is the transformation matrix from the local to the global system.

Equation (16) represents the rigid base condition. If base flexibility is incorporated, then equation (16) becomes

\[
\{\Delta\} = [T]^{-1}\{\lambda\} + \begin{bmatrix} u_0 \\ v_0 \\ 0 \end{bmatrix}
\] (17)

where \(u_0\) and \(v_0\) are the additional global displacements in the \(x\)- and \(y\)-directions, respectively, which are created by the base flexibility. At height \(h\) above the base, the additional displacements are related to the base rotation components by

\[
u_0 = h\beta_x \quad \text{and} \quad v_0 = h\beta_y
\] (18)

5. NUMERICAL EXAMPLES

5.1. Monosymmetric structure

A sixteen storey monosymmetric structure (Figure 6), with a storey height of 3.0 m, was analysed by Gluck\(^{11}\) for a uniform lateral load \(q = 40 \text{ kN m}^{-1}\) in the \(y\)-direction. The stiffening elements consist of an arrangement of frames and shear walls, spanning the \(x\)- and \(y\)-directions, with the geometric properties listed in Figure 6. This example has also been extended by

Figure 6. Structural plan and material properties of the building
Swaddiwudhipong et al. to the case of variable rotational foundation stiffness. Several other authors (Coull and Abdul Wahab, Sternik and Gluck) have also used this example for different purposes.

As the structure is asymmetric in plan, the lateral loading will result in both bending and torsional effects, both of which have been investigated. The results using the discrete force method are compared with results from the continuous connection analyses given in References 5 and 11–13. The distributions of lateral deflection, the bending moment of wall 1, and the torsional rotation of the structure are shown in Figure 7. From the results obtained it is seen that the curves correspond very closely to the continuous connection solution.

In this example, the effect of an elastic foundation on the behaviour of the structure is also studied and compared with Swaddiwudhipong et al. who only considered rotational foundation stiffness. The variation of the deflection profile at wall 1 due to the different values of foundation stiffness is shown in Figure 8(a). Here, also, there is good agreement between the discrete force method and the continuum approach used by Swaddiwudhipong et al. Figure 8(b) shows the torsion angle distribution obtained for different soil stiffnesses and it can be seen that for a rigid base foundation (Figure 7(b)), the twist angle over the height changes direction. This occurs because the torsional stiffness close to the base is governed by the wall system, whereas the frame system predominates higher up the structure. Since the centres of twist of these two systems are on opposite sides of the y-axis, it follows that the torsional rotation of the structure will reverse. Confirmation of this behaviour may be obtained from Figure 7(c) which shows the bending moment changing slope, with a resulting change in the direction of the wall's shearing force, around the change in rotation level (Figure 7(b), storey 12).

The effect of foundation stiffness on the deflection of the structure is shown in Figure 8(a). It can be seen that the deflection increases significantly as the foundation stiffness decreases. Also, the bending moments in wall 1 (Figure 8(c)) decrease considerably with decreasing stiffness base flexibility.

Foundation flexibility affects the overall behaviour of the structure significantly. It can be seen from Figure 8(b), for example, that the building tends to rotate to one side only if the base

![Figure 7. (a) Lateral deflection; (b) torsional angle; (c) moment in wall 1](image-url)
flexibility is reduced. This reflects the fact that the reduced bending, and hence shear, in the wall system results in the frame torsional stiffness predominating over the full height.

5.2. Partially closed section

A twenty storeys partially closed structure (Figure 9), subjected to uniform distributed load $q = 10 \text{ kN} \cdot \text{m}^{-1}$, has been analysed by Stafford Smith and Coull.\textsuperscript{14} The structure consists of 3.5 m height storey with the plan arrangement and material properties shown in Figures 9(a) and 9(b). The effects of soil interaction on the structure have been investigated using the discrete force approach. In the discrete force method, the structure is divided into elements (walls) linked by modes (joints) which, together with the orientation of each individual wall, are shown in Figure 9(b).

Table I shows the global deflections and the rotational angle at point A (Figure 9(b)) at the top of the structure. Good agreement between the discrete force method and the other methods listed in Table I is achieved when rigid base conditions apply. The vertical direct stresses at the base are shown in Figure 9(c) and correlate well with the other solutions.

The results are presented in Tables II, III and IV, in which the global deflections and rotations at the top are given. The deflection increases at the top but the rotation decreases when the soil flexibility is reduced below values typical of those encountered in practice. These low foundation flexibilities have been investigated in order that the robustness of the analysis could be verified over a wide flexibility range. Tables II–IV show good correlation between the discrete force method and frame analogy method when either axial or rotational flexibility, or both, are considered, for variable values of soil stiffness.

The distributions of lateral deflection and the angle of rotation of the structure are shown
Figure 9. (a) Core subjected to uniform distributed load; (b) building plans and idealized model used by the discrete force method; (c) stress distribution at ground floor (all stresses are in kilonewtons per square metre)
Table I. Rotation and deflection at the top of the core

<table>
<thead>
<tr>
<th>Method of analysis</th>
<th>Top deflection at point A (mm)</th>
<th>Global rotation at the top (rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCM</td>
<td>0.175</td>
<td>0.0080</td>
</tr>
<tr>
<td>BFA</td>
<td>0.180</td>
<td>0.0078</td>
</tr>
<tr>
<td>DFM</td>
<td>0.175</td>
<td>0.0080</td>
</tr>
</tbody>
</table>

BFA: braced frame analogy; DFM: discrete force method; CCM: continuous connection method.

Table II. Stafford Smith's example (1991) when axial springs are considered

<table>
<thead>
<tr>
<th>Method of analysis</th>
<th>Type of soil</th>
<th>Axial spring (kN m⁻¹)</th>
<th>Top deflection (mm)</th>
<th>Top rotation (rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DFM</td>
<td>Hard rock</td>
<td>∞</td>
<td>97.10</td>
<td>0.0080</td>
</tr>
<tr>
<td>BFA</td>
<td>Hard rock</td>
<td>∞</td>
<td>102.20</td>
<td>0.0078</td>
</tr>
<tr>
<td>DFM</td>
<td>Dense sand</td>
<td>7.5 × 10⁸</td>
<td>97.14</td>
<td>0.0080</td>
</tr>
<tr>
<td>BFA</td>
<td>Dense sand</td>
<td>7.5 × 10⁸</td>
<td>102.30</td>
<td>0.0078</td>
</tr>
<tr>
<td>DFM</td>
<td>Stiff Clay</td>
<td>1.2 × 10⁷</td>
<td>98.54</td>
<td>0.0080</td>
</tr>
<tr>
<td>BFA</td>
<td>Stiff clay</td>
<td>1.2 × 10⁷</td>
<td>103.12</td>
<td>0.0078</td>
</tr>
<tr>
<td>DFM</td>
<td>Weak soil</td>
<td>3.0 × 10⁶</td>
<td>100.14</td>
<td>0.0080</td>
</tr>
<tr>
<td>BFA</td>
<td>Weak soil</td>
<td>3.0 × 10⁶</td>
<td>104.51</td>
<td>0.0078</td>
</tr>
<tr>
<td>DFM</td>
<td>Poor soil</td>
<td>3.0 × 10⁵</td>
<td>112.75</td>
<td>0.0080</td>
</tr>
<tr>
<td>BFA</td>
<td>Poor soil</td>
<td>3.0 × 10⁵</td>
<td>116.10</td>
<td>0.0078</td>
</tr>
<tr>
<td>DFM</td>
<td>—</td>
<td>3.0 × 10³</td>
<td>147.60</td>
<td>0.0054</td>
</tr>
<tr>
<td>BFA</td>
<td>—</td>
<td>3.0 × 10³</td>
<td>159.67</td>
<td>0.0048</td>
</tr>
<tr>
<td>DFM</td>
<td>—</td>
<td>3.0 × 10³</td>
<td>149.32</td>
<td>0.0053</td>
</tr>
<tr>
<td>BFA</td>
<td>—</td>
<td>3.0 × 10³</td>
<td>163.96</td>
<td>0.0045</td>
</tr>
<tr>
<td>DFM</td>
<td>—</td>
<td>2.0 × 10²</td>
<td>149.54</td>
<td>0.0053</td>
</tr>
<tr>
<td>BFA</td>
<td>—</td>
<td>2.0 × 10²</td>
<td>164.45</td>
<td>0.0045</td>
</tr>
</tbody>
</table>

BFA: braced frame analogy; DFM: discrete force method; CCM: continuous connection method.

The variation of the deflection profile of the partially closed section due to the different values of foundation stiffness is shown in Figure 10(a). It can be seen from Figure 10(a) that hard rock, dense sand and stiff clay behave like a rigid foundation. When the foundation stiffness is poor or not practical, the deflection increases up to 35% as compared to a rigid foundation.

The torsion angle distribution obtained for the different soil stiffnesses is shown in Figure 10(b), in which it can be seen that increased soil flexibility actually leads to a reduction in the twist and at two higher levels of the core. Figure 11 shows the distribution of the vertical stresses at the base due to the variable values of foundation stiffness.

Poor soil (Figure 11(e)) is to be avoided for such a structure, because the stresses are concentrated in one wall (Figure 9(b), wall 3). Also, it can be seen from Figures 11(a) and 11(e) that the flexible foundation affects mainly the stresses in the lower portion of the core and, in general, has negligible effect on the stresses in the upper portion of the structure, a feature that has been commented in previously.¹⁰
### Table III. Stafford Smith’s example (1991) when rotational springs are considered

<table>
<thead>
<tr>
<th>Method of analysis</th>
<th>Type of soil</th>
<th>Rot. Spring (kNm rad(^{-1}))</th>
<th>Top deflection (mm)</th>
<th>Top rotation (rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DFM</td>
<td>Hard rock</td>
<td>(\infty)</td>
<td>97.10</td>
<td>0.0080</td>
</tr>
<tr>
<td>BFA</td>
<td>Hard rock</td>
<td>(\infty)</td>
<td>102.20</td>
<td>0.0078</td>
</tr>
<tr>
<td>DFM</td>
<td>Dense sand</td>
<td>(1.0 \times 10^{12})</td>
<td>97.13</td>
<td>0.0080</td>
</tr>
<tr>
<td>BFA</td>
<td>Dense sand</td>
<td>(1.0 \times 10^{12})</td>
<td>102.40</td>
<td>0.0078</td>
</tr>
<tr>
<td>DFM</td>
<td>Stiff clay</td>
<td>(1.0 \times 10^{10})</td>
<td>97.24</td>
<td>0.0080</td>
</tr>
<tr>
<td>BFA</td>
<td>Stiff clay</td>
<td>(1.0 \times 10^{10})</td>
<td>102.44</td>
<td>0.0078</td>
</tr>
<tr>
<td>DFM</td>
<td>Weak soil</td>
<td>(1.0 \times 10^{9})</td>
<td>97.33</td>
<td>0.0080</td>
</tr>
<tr>
<td>BFA</td>
<td>Weak soil</td>
<td>(1.0 \times 10^{9})</td>
<td>102.44</td>
<td>0.0078</td>
</tr>
<tr>
<td>DFM</td>
<td>Poor soil</td>
<td>(5.0 \times 10^{8})</td>
<td>97.53</td>
<td>0.0080</td>
</tr>
<tr>
<td>BFA</td>
<td>Poor soil</td>
<td>(5.0 \times 10^{8})</td>
<td>102.50</td>
<td>0.0078</td>
</tr>
<tr>
<td>DFM</td>
<td>—</td>
<td>(2.5 \times 10^{8})</td>
<td>97.60</td>
<td>0.0080</td>
</tr>
<tr>
<td>BFA</td>
<td>—</td>
<td>(2.5 \times 10^{8})</td>
<td>102.67</td>
<td>0.0078</td>
</tr>
<tr>
<td>DFM</td>
<td>—</td>
<td>(1.0 \times 10^{8})</td>
<td>98.50</td>
<td>0.0080</td>
</tr>
<tr>
<td>BFA</td>
<td>—</td>
<td>(1.0 \times 10^{8})</td>
<td>102.68</td>
<td>0.0078</td>
</tr>
</tbody>
</table>

DFM: discrete force method; BFA: braced frame analogy.

### Table IV. Stafford Smith’s example (1991) when both rotational and axial springs are considered

<table>
<thead>
<tr>
<th>Method of analysis</th>
<th>Type of soil</th>
<th>Rot. spring (kNm rad(^{-1}))</th>
<th>Axial spring (kN m(^{-1}))</th>
<th>Top deflection (mm)</th>
<th>Top rotation (rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DFM</td>
<td>Hard rock</td>
<td>(\infty)</td>
<td>(\infty)</td>
<td>97.10</td>
<td>0.0080</td>
</tr>
<tr>
<td>BFA</td>
<td>Hard rock</td>
<td>(\infty)</td>
<td>(\infty)</td>
<td>102.20</td>
<td>0.0078</td>
</tr>
<tr>
<td>DFM</td>
<td>Dense sand</td>
<td>(1.0 \times 10^{12})</td>
<td>(7.5 \times 10^{8})</td>
<td>97.18</td>
<td>0.0080</td>
</tr>
<tr>
<td>BFA</td>
<td>Dense sand</td>
<td>(1.0 \times 10^{12})</td>
<td>(7.5 \times 10^{8})</td>
<td>104.30</td>
<td>0.0078</td>
</tr>
<tr>
<td>DFM</td>
<td>Stiff clay</td>
<td>(1.0 \times 10^{10})</td>
<td>(1.2 \times 10^{7})</td>
<td>100.20</td>
<td>0.0080</td>
</tr>
<tr>
<td>BFA</td>
<td>Stiff clay</td>
<td>(1.0 \times 10^{10})</td>
<td>(1.2 \times 10^{7})</td>
<td>104.80</td>
<td>0.0078</td>
</tr>
<tr>
<td>DFM</td>
<td>Weak soil</td>
<td>(1.0 \times 10^{8})</td>
<td>(3.0 \times 10^{6})</td>
<td>106.97</td>
<td>0.0078</td>
</tr>
<tr>
<td>BFA</td>
<td>Weak soil</td>
<td>(1.0 \times 10^{8})</td>
<td>(3.0 \times 10^{6})</td>
<td>105.31</td>
<td>0.0078</td>
</tr>
<tr>
<td>DFM</td>
<td>Poor soil</td>
<td>(5.0 \times 10^{8})</td>
<td>(3.0 \times 10^{5})</td>
<td>131.66</td>
<td>0.0073</td>
</tr>
<tr>
<td>BFA</td>
<td>Poor soil</td>
<td>(5.0 \times 10^{8})</td>
<td>(3.0 \times 10^{5})</td>
<td>128.60</td>
<td>0.0076</td>
</tr>
<tr>
<td>DFM</td>
<td>—</td>
<td>(2.5 \times 10^{8})</td>
<td>(3.0 \times 10^{3})</td>
<td>159.10</td>
<td>0.0063</td>
</tr>
<tr>
<td>BFA</td>
<td>—</td>
<td>(2.5 \times 10^{8})</td>
<td>(3.0 \times 10^{3})</td>
<td>162.23</td>
<td>0.0072</td>
</tr>
<tr>
<td>DFM</td>
<td>—</td>
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<td>(3.0 \times 10^{2})</td>
<td>174.60</td>
<td>0.0030</td>
</tr>
<tr>
<td>BFA</td>
<td>—</td>
<td>(1.0 \times 10^{8})</td>
<td>(3.0 \times 10^{2})</td>
<td>183.27</td>
<td>0.0040</td>
</tr>
</tbody>
</table>

DFM: discrete force method; BFA: braced frame analogy.

5.3. Shear walls and core assembly

A 27-storey RC building located in Cincinnati (Ohio) was investigated by Somaprasad and Aktan\(^{15}\) to evaluate seismic vulnerability. The building was closed in 1978 because of inadequate fire safety measures, and it was imploded on 23 June 1991. A typical structural plan is shown in Figure 12(a). The typical floor system consisted of an 184 mm thick cast-in-place light RC flat slab supported by normal weight RC columns, walls and cores. The total height of the building was 84.8 m with a typical storey height of 2.74 m (five storeys were of other
heights, as shown in the elevation in Figure 12(b)). The primary lateral force resisting system considered of two central cores, peripheral shear walls and the slab-column frames. Core walls were 305 mm thick and peripheral shear walls were 203 mm thick. The cores were coupled by a 127 mm thick floor slab.

The foundation response was measured by experimental tests under a loading of 1 MN at the 26th floor in the N–S direction. The displacements of the column foundations, as shown in Figure 13, were used to calibrate the axial spring stiffnesses. This real building was modelled by the discrete force method to compare the results with the experimental test and an analytical model using the finite element method.

Figure 14 illustrates the idealized model used in the discrete force method by dividing the structure into 22 walls and 26 joints, with the orientation of each individual wall being shown. To estimate the values of the axial stiffnesses, the discrete force method has been used with a rigid base assumption to get the values of the axial forces applied to the walls which are subjected to the lateral loading. With the measured displacements provided by the test and the normal forces given by the discrete force method previously, the axial stiffness flexibility $K_V$ can be calculated the simple formula $K_V = N/d_v$, in which $N$ is the normal axial force at the base and $d_v$ is the vertical displacement.

Comparison with the experimental test values indicated that E–W loading results correlated
well with the discrete force method, the finite element and the measured value; under N–S loading, the discrete force displacement values were 35% larger than the measured values shown in Figure 15 when the stiffness of the foundation was considered. The finite element analysis produced a similar discrepancy and Somaprasad and Aktan\(^{15}\) concluded that this was due to the omission of the effects of floor slab action in the region between the cores. This slab in the region serves to couple the two cores. This action was modelled in the discrete force method by significantly increasing the stiffness of the N–S beams that connect the two cores. With this amendment, the discrete force analysis compared well (Figure 15(a)) with both the test results and also with finite element results when modified for core coupling. Figure 15 shows that the behaviour of the building is not significantly affected by the base flexibility because the foundation is built on hard rock, which can be considered a rigid base, as concluded in Example 2.

Figure 11. Stress distribution due to axial and rotational foundation flexibility: (a) rigid base; (b) hard rock; (c) dense sand; (d) stiff clay; (e) very weak soil (all stresses are in kilonewtons per square metre) discrete force method. • braced frame analogy
Figure 12. (a) Typical floor plan; (b) east elevation of building
Figure 13. Measured foundation displacements corresponding to lateral force of 1 MN at floor 26

Figure 14. The idealized model used by the discrete force method

Figure 15. Measured and compared lateral deflection: (a) N–S direction; (b) E–W direction
6. CONCLUSIONS

Based on the discrete force method of analysis, an analysis has been presented for three dimensional complex shear walls resisting on flexible foundations and subjected to any type of lateral loading. Both the vertical and rotational stiffnesses of the foundation were taken into account in the analysis. The soil and structure were coupled by imposing equilibrium and compatibility conditions along the discrete force method interfaces. Based on past studies, the discrete force method has demonstrated the capability of accurately modelling the linear behaviour of complicated structures with dramatic computational savings. The system model developed here offers the potential for the same sort of computational savings in the analysis of the entire building soil system.

Typical examples show that foundation flexibility can have an important effect on the behaviour of coupled shear walls or shear walls and core assemblies. The flexible foundation affects mainly the stresses at the lower portion of shear walls and, in general, has negligible effect on the stresses at the upper portion of the structure. For practical purposes, foundations built on hard rock or dense sand can be considered as rigid.

The examples presented in this work clearly demonstrate that the proposed discrete force method is well suited to investigate more complicated soil structure interaction problems. The method is highly flexible and its results are in excellent agreement when compared with solutions obtained with the finite element, the braced frame analogy, the continuous connection method and experimental tests.

REFERENCES