Soil–structure interaction in tall buildings by a discrete force method

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The importance of base flexibility on the elasto-plastic behaviour of spatial shear walls subjected to any type of loading was examined. An analysis of elasto-plastic spatial shear walls was developed using a discrete force method, which models the shear walls as a system of interconnected discrete structural elements and the base flexibility by effective rotational and vertical elastic stiffnesses $K_v$ and $K_b$, respectively. As the magnitudes of these stiffnesses depend on the properties of the supporting soil and also on the characteristics of the foundation itself, different soils were considered. The analysis was based on the force approach, using as redundants the shear forces, not only at the contraflexural points of the connecting beams, but also at wall junctions. The elasto-plastic condition was restricted to the connecting beams by using a convenient bilinear model for the force formulation. The objective was to provide a theoretical treatment of the foundation–structure interaction that is suitable for implementation in the discrete force method. In addition, the results obtained in the discrete force modelling of the soil interaction are presented, and the applicability of the model to some representative problems is discussed.

Keywords: buildings; research & development; structure & design

Introduction
The first study of the effect of flexible bases was reported by Coull, who treated rotational and vertical base flexibility separately, employing the continuum approach, and then superimposed the results to assess the general case of coupled rotational and vertical base boundary conditions. In a subsequent study, Tso and Chan examined simultaneous vertical and rotational movement at the bases of the coupled walls. Pekau and Cistera examined the importance of base flexibility on the non-linear behaviour of coupled shear walls subjected to pseudo-static lateral loading using the classic continuum method. Only a few papers have dealt with the analysis of three-dimensional soil–structure interaction problems. Swaddiwudhipong et al. were the first to investigate three-dimensional flexible foundations using the continuous connection method. Spyrakos and Beskos were among the first to investigate two- and three-dimensional flexible foundations, modelled with finite elements and coupled to half-spaces represented by boundary elements.

1. The discrete force approach to shear wall analysis has been presented previously for the linear elastic analysis of planar walls and for spatial shear wall systems. The method has been extended to the elasto-plastic condition, by assuming that plasticity is restricted to the connecting beams. The method has been developed further for the elastic and elasto-plastic analysis of planar coupled shear walls with flexible bases. The method involves the assumption that there are contraflexural points at the midpoints of beams connecting co-planar walls and at the junctions of non-planar walls or non-planar beam/wall intersections (Fig. 1). The system of contraflexural points established in this way divides the shear wall system into a number of individual wall components, which may have beam stubs on one or both sides (Fig. 1(b)).

2. The need to design for ductile behaviour of spatial shear wall structures if they are to survive strong earthquake ground motions points to the need to examine the elasto-plastic response of such systems. Thus, we present here the results of a study of elasto-plastic spatial shear walls resting on flexible bases, in which coupling beams are permitted to yield while the walls and the supporting soil are assumed to remain elastic. The discrete nature of the force approach, however, offers almost total flexibility in terms of wall-plan arrangement and elevational irregularities.

Elastic analysis

4. If the system is divided into a number of wall components by the contraflexural points assumption described earlier, then the introduction of releases at these points will require the determination of a redundant shear force at each release. No redundant moments or axial loads will be present, the former due to the contraflexural assumption and the latter due to the assumed presence of rigid in-plane floors at each beam level. The full set of redundant shear forces $q_i$ may be determined by the enforcement of the compatibility on the relative vertical displacements of adjacent released...
positions. The form of the compatibility requirement varies according to whether the condition is applied between two upper storeys or between the first floor and the base. These two situations are therefore described separately.

**Compatibility condition between upper storeys**

5. Figure 2 shows a typical storey at a general joint which includes connecting beam stubs, although these may be eliminated by setting \( a \) (the appropriate stub length/s) to zero. For beams \( s \) and \( s - 1 \), which bound storey \( s \), the vertical separation of the cut position of beam \( s \) and that of beam \( s - 1 \) must be identical, regardless of whether this displacement is evaluated from the left-hand (1) or the right-hand (2) side. If the vertical distance between the cut points measured from side 1 is \( h_s \), then:

\[
h_s = h_i + \delta_{1a} + \delta_{1b} + \delta_{1c} + \delta_{1d} + \delta_{1e}
\]

In equation (1), the displacements \( \delta_{1a-e} \) are due to: (a) wall axial extension, (b) wall bending deformation, (c) connecting beam bending deformation, (d) connecting beam shear deformation, and (e) wall shear deformation. These displacements may all be expressed in terms of the redundant shears \( \{q\} \) and the applied loading, the appropriate expressions for which have been reported previously.6

6. By a similar argument, for side 2:

\[
h_s = h_i + \delta_{2a} + \delta_{2b} - \delta_{2c} - \delta_{2d} - \delta_{2e}
\]

For deformation compatibility between the adjacent cuts, \( h_s = h_s \), or:

\[
(\delta_{1a} - \delta_{2a}) + (\delta_{1b} + \delta_{2b}) + (\delta_{1c} + \delta_{2c}) + (\delta_{1d} + \delta_{2d}) = (\delta_{1b} + \delta_{2b})
\]

**Flexible bases**

7. When the compatibility condition involves the base level it is necessary to consider the rotation of the walls and the relative base settlement, which are directly proportional to the bending moment and axial force, respectively, at the base of the wall. The stiffnesses of the flexible foundation are represented by translational and rotational springs with stiffness, \( K_v \), and \( K_b \) at the base of each wall (Fig. 3). In the elasto-plastic range of behaviour, it is assumed that the walls and supporting soil remain elastic.

**Elastic vertical movement**

8. In this case it is assumed that the walls remain vertical but, owing to the elasticity of the foundations, a vertical displacement occurs which is directly proportional to the magnitude of the axial force at the base of the wall (Fig. 4(a)). The vertical displacements due to foundation movements, for walls 1 and 2, \( \delta_{v1} \) and \( \delta_{v2} \), must therefore be taken into account when forming the compatibility equation at joint 1 for storey 1, and may be evaluated as

\[
\delta_{v1} = \frac{N_{11}}{K_v}
\]

and

\[
\delta_{v2} = \frac{N_{21}}{K_v}
\]
Elastic rotational movement

9. In this case it is assumed that the foundations rotate under the influence of the moments at the wall bases. As a result of a fundamental assumption that the walls rotate equally, because the base level rotates as a rigid body, the components of base rotation $\beta_x$ and $\beta_y$ will be proportional to the base moments and, for an elastic foundation, may be expressed as

$$\beta_x = \frac{m_{xo}}{K_{b1}}$$

and

$$\beta_y = \frac{m_{yo}}{K_{b2}}$$

where $m_{xo}$, $m_{yo}$ are the moments at the base in the $x$ and $y$ directions, respectively, and $K_{b1}$, $K_{b2}$ are the rotational stiffnesses of the foundation system in the $x$ and $y$ directions, respectively. The rotation of walls 1 and 2 (Fig. 4(b)) is then given by

$$\{ \beta_1 \} = \begin{bmatrix} c_1 & s_1 \\ c_2 & s_2 \end{bmatrix} \{ \beta \}$$

where $c$ and $s$ are the cosine and sine of the orientation of the walls. Thus, from Fig. 4(b), the additional movements due to base rotation are given by

$$\delta_{b1} = c_{1j1} \beta_1$$

and

$$\delta_{b2} = c_{2j1} \beta_2$$

Base compatibility condition

10. If the movements due to base flexibility are taken into account, then equation (1) for the vertical distance between ground level and the first storey cut as measured from side 1 becomes

$$h_{1z} = h_i + \delta_{1a} - \delta_{1b} + \delta_{1c} + \delta_{1d} + \delta_{v1} - \delta_{f1}$$

By similar argument, for side 2

$$h_{2z} = h_i + \delta_{2a} + \delta_{2b} - \delta_{2c} - \delta_{2d} + \delta_{v2} + \delta_{f2}$$

For deformation compatibility between the adjacent cuts:

$$\left( \delta_{1a} - \delta_{2a} \right) + \left( \delta_{1c} + \delta_{2c} \right) + \left( \delta_{1d} + \delta_{2d} \right) + \left( \delta_{v1} - \delta_{v2} \right) + \left( \delta_{f1} + \delta_{f2} \right) = \left( \delta_{1b} + \delta_{2b} \right)$$

Stress resultant solution

11. By expressing a complete set of compatibility conditions in terms of the redundant shears and the applied loading, a set of flexibility equations is obtained:

$$[F][q] = [r]$$

The vector $[r]$ is dependent on the applied lateral loading, which can be evaluated analytically or numerically, and the equations may be solved to obtain the redundant shears:

$$[q] = [F]^{-1}[r]$$

The redundant shears may then be used to obtain the stress resultant response and, subsequently, the displacement profile, for the complete wall system.8

Elasto-plastic analysis

Connecting beam response

12. The deformation of the coupling beams is a combination of flexural and shear deformation. The flexural deformation, in which the beam bends in double curvature with a point of contraflexure at the centre of the span and the associated forces, is shown in Fig. 5(a). The
action of the shear force acting through the point of contraflexure produces maximum bending moments at the wall supports, with the consequent development of flexural cracks. When the applied shear force is increased, the flexural cracking progresses towards the compression corners. Eventually, crushing will take place in the compression corners, resulting in the ultimate failure of the beam (Fig. 5(b)). In extending the discrete force formulation to the elasto-plastic condition, it is assumed that plasticity is restricted to the connecting beams. This assumption is usually warranted since, in practice, flexible connecting beams are desirable for both dynamic response and ductile failure mode considerations. However, several studies\textsuperscript{14–16} have shown that plasticity imposes

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig4}
\caption{Vertical and horizontal settlement at the base. \(j\), joint number}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig5}
\caption{Lintel beam:
(a) deformation;
(b) cracking pattern}
\end{figure}
a severe ductility requirement on the connecting beams which is unlikely to be fulfilled in
practice. Accordingly, it has been suggested\textsuperscript{17,18} that the analysis should be terminated if a
specified beam ductility limit is exceeded. Such a procedure has not been adopted here, but
could be incorporated if so desired. In common
with most previous investigations, perfectly
elastic/perfectly plastic behaviour is assumed
for the connecting beams, since the adoption
of more complex models has been shown\textsuperscript{19} not
to have a major effect on the elasto-plastic
response. The bilinear model is particularly
convenient for the force formulation, as the
perfectly plastic assumption results in the
connecting beam shear forces remaining
constant once their elastic limit has been
exceeded, and hence the number of redundants
to be determined is progressively decreased.

Analysis

13. Following an initial elastic analysis, the
most severely loaded connecting beam $i$ is
identified and the load factor $\lambda_1$ at which this
beam reaches its plastic shear value $Q_{ul}$ is
obtained by linear extrapolation. As the shear
force in beam $i$ subsequently remains constant
at $Q_{ul}$, the unknown $Q_i$ may be deleted from \{$q_1$\}
and the compatibility equation relating
to storey $i$ becomes both superfluous and
inappropriate in view of the subsequent plastic
deformation of beam $i$. Also, the compatibility
equation for storey $i+1$ will now refer to the
compatibility of deformations between
succeeding storeys $i+1$ and $i-1$ (Fig. 6(a)) rather than to
storey $i+1$ alone. This modification may be
effected if the flexibilities previously associated
with storey $i$ are accrued to those of storey $i+1$.

14. The analysis is now repeated. As the
process is linear, at any stage $s$ the load factor $\lambda_s$
may be obtained by accumulation of the pre-
vious factor $\lambda_{s-1}$, with an addition, obtained by
extrapolation, required to increase the most
severely loaded beam $i$ to its ultimate shear
capacity. It is necessary to keep a note of the
beams which have achieved their plastic capa-
city, since at a general stage (Fig. 6(b)) several
beams at their ultimate load may intervene
between beam $i$ and the nearest superior $j$ and
lower $k$ beams, which are still elastic. The prin-
ciple remains the same, however, in that the

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig6.png}
\caption{Spread of plasticity: (a) isolated beam; (b) within a plastic zone}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig7.png}
\caption{Coupling beams: (a) constant stiffness; (b) stiff at the top; (c) stiff at levels 5–7 and top
floor. (d) Building plans and model used in the discrete force method}
\end{figure}
addition to storey \( j \) of the flexibilities presently associated with \( j \) will ensure that the compatibility condition for storey \( j \) subsequently refers to deformations between storey levels \( j \) and \( k \). The complete procedure is repeated until all the connecting beams have achieved their ultimate capacity, at which stage the system is statically determinate. Thus, this force approach becomes simpler as plasticity spreads, rather than requiring additional plastic rotation variables as is the case with the stiffness method.

**Computational aspects**

15. The flexibility matrix \( [F] \) of equation (6) is not symmetric, but can be arranged to possess an extremely sparsely populated lower triangle, provided that an appropriate sequencing of the connecting beams is adopted. The preferred sequencing is to order the connecting beams storey by storey. If this ordering is used, then the band for the forward elimination process is limited to the maximum number of beams at any storey level. The band for the backward elimination process can approach the full size of the matrix. However, as the forward elimination is the time-consuming operation, the suggested arrangement results in a very efficient computational process, which is of importance in an elasto-plastic analysis where repeated solutions are needed.

16. Also of importance with respect to computational efficiency is the reduction in the number of unknowns which occurs as the plasticity develops. As noted above, each solution in the elasto-plastic analysis results in one more beam attaining its constant ultimate shear load and hence becoming a predetermined quantity for subsequent analyses. Once plasticity has spread to a significant number of connecting beams, the consequent reduction in the effective size of \( (q) \) markedly improves the efficiency of the equation solution process.

### Numerical example

**Partially closed section**

17. A 20-storey partially closed structure (Fig. 7), subjected to uniform distributed load \( w = 10 \text{kN/m} \), has been analysed by Stafford Smith and Coull. \(^{20}\) The structure consists of 3.5 m high storeys with the plan arrangement and material properties shown in Fig. 7. The effects of soil interaction on the structure have been investigated using the discrete force approach. In the discrete force method, the structure is divided into elements (walls) linked by nodes (joints) which, together with the orientation of each individual wall, are shown in Fig. 7(d).

18. The core has been analysed previously in the elastic range by Nadjai and Johnson \(^{11}\) for different foundation stiffnesses. In the present investigation, elasto-plastic behaviour was included. The effects of the soil–structure interaction were investigated for three cases under the following conditions:

- (a) Case 1 (Fig. 7(a)): the connecting beams have a constant ultimate shear capacity of \( Q_u = 200 \text{kN} \).
- (b) Case 2 (Fig. 7(b)): the top beam is stiffened, to increase the structural efficiency, to an ultimate shear capacity \( Q_u = 2400 \text{kN} \).
- (c) Case 3 (Fig. 7(c)): the same shear capacity as for cases 1 and 2, but the beams at storey levels 5, 6 and 7 were stiffened to an ultimate shear capacity of \( Q_u = 1000 \text{kN} \).

The beams at storey levels 5, 6 and 7 were stiffened because they are situated at the third height of the structure from the base, which is a critical zone where plasticity almost always occurs first.

19. The load–displacement curves for the four different base stiffnesses are shown in Fig. 8. The data are plotted up to the ultimate state where plastification of all the coupling beams has occurred. It can be seen that varying

### Table 1. Results for spatial shear walls of different footing stiffnesses: discrete force method

<table>
<thead>
<tr>
<th>Case</th>
<th>Axial stiffness ( k \text{N/m} )</th>
<th>Rotation stiffness: ( k \text{N/m/rad} )</th>
<th>Yield deflection: ( \text{mm} )</th>
<th>Ultimate deflection: ( \text{mm} )</th>
<th>Yield load: ( k \text{N/m} )</th>
<th>Ultimate load ( k \text{N/m} )</th>
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<td>32 02</td>
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<td>( 3.0 \times 10^5 )</td>
<td>202</td>
<td>419</td>
<td>17 41</td>
<td>30 00</td>
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<td></td>
<td>( 8.0 \times 10^4 )</td>
<td>( 1.2 \times 10^5 )</td>
<td>271</td>
<td>537</td>
<td>16 77</td>
<td>29 22</td>
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<tr>
<td></td>
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<td>584</td>
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<td>28 64</td>
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<tr>
<td>2</td>
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<td>( \infty )</td>
<td>173</td>
<td>683</td>
<td>18 01</td>
<td>56 22</td>
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<td>( 3.0 \times 10^5 )</td>
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<td>756</td>
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<td>53 87</td>
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<td>52 42</td>
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<td>16 46</td>
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<td>( \infty )</td>
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<td>1031</td>
<td>18 01</td>
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<td>18 10</td>
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<td>1545</td>
<td>16 48</td>
<td>68 22</td>
</tr>
</tbody>
</table>
the base conditions from sound rock to footings on dense sands results in a reduction of only 12% in the ultimate load for the three cases. However, the top deflection is much more sensitive to the base stiffness, showing a corresponding increase of 52% for the three cases (Table 1). It is interesting to note that the first yield of the coupling beams occurs at approximately the same load regardless of the base flexibility. It is thus clear that foundation flexibility affects primarily the lateral stiffness of the coupled walls, having a relatively small influence on lateral strength.

20. It is also evident that from Fig. 8 and Table 1 that the presence of a stiff top beam (case 2) increases from the ultimate load to 43%
compared to case 1. For case 3, the ultimate load is increased by 57% and 25% compared to cases 1 and 2, respectively.

21. Plots showing the loads at which the various connecting beams reach their plastic capacities for the different cases considered are given in Fig. 9. For case 1, with sound rock, for example, the first beam to reach its plastic capacity is at storey 6 (height 22 m) at a load of approximately 18 kN/m. At higher loading levels, the vertical intercepts on the plots in Fig. 9(a) will define a plastic zone in which all the connection beams have yielded, but the beams outside this region will still be responding elastically. From the shape of the case 1 (sound rock) plot (Fig. 9(a)) it can be seen that, following the yield at storey 6, plasticity initially starts to spread downwards, but then begins to extend rapidly towards the top of the core. Full yield is attained by the lowest and highest beams reaching their plastic capacities almost simultaneously. Increased base flexibility changes this behaviour in that the initial spread of plasticity downwards is accentuated and

![Fig. 9. Variation in the plastic zone for different separate footing stiffnesses: (a) constant beam stiffness; (b) stiff beam at the top storey; (d) stiff beams at levels 5–7 and the top storey](image)
includes the lowest beam well before the beams in the upper half of the core have reached their plastic capacity.

22. In case 2, increased base flexibility again encourages early spread of plasticity towards the base, and the presence of the strengthened top beam ensures that the development of plasticity towards the top of the core is delayed, even in the sound rock situation.

23. The presence of intermediate beam strengthening in case 3 alters the plastic response, as two plastic zones are created, one on either side of the strengthened beams, which remain elastic until approximately 75% of the ultimate load has been reached. Nevertheless, the effect of the strengthened top beam is sufficiently pronounced to ensure that it is the final beam to reach its capacity in all cases.

Conclusions

24. Based on the discrete force method of analysis, an analysis has been presented for the elasto-plastic analysis of spatial shear walls systems on flexible foundations and subjected to any type of loading which can be met in practice. Both foundation vertical stiffness and rotational stiffness are taken into account in the analysis. The soil and structure were coupled by imposing equilibrium and compatibility conditions along the discrete force method interfaces. The system model developed here offers the potential for significant computational savings in the analysis of an entire building soil system.

25. A typical example showed that foundation flexibility can have an important effect on the behaviour of core shear walls, and it may be concluded that:

(a) Stiffening the top beam and critical beams in the structure can produce considerable enhancement of the collapse load.

(b) The spread of plasticity becomes more rapid and the post-yield capacity is thereby somewhat reduced when foundation flexibility increases. Furthermore, as expected, deflection increases significantly with foundation flexibility.

(c) Foundation flexibility affects primarily the lateral stiffness of the coupled walls, the influence on lateral strength being relatively small.

The example presented in this work clearly demonstrates that the proposed discrete force method is well suited to investigating more complicated soil-structure interaction problems.

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