Collapse of infilled steel frames with semi-rigid connections

A. Nadjai, BEng, MSc, PhD, MStructE, PGCU, and P. Kirby, BEng, PhD

Introduction

Wood\(^1\) noted that, in a number of his tests, premature failure of joints occurred, indicating that the connections were weaker than the members that they were joining. May and Ma\(^2\) extended this work to cover cases in which the ultimate moment capacity of the joints was lower than that of the connected members for precast concrete frames. The present work is an extension of May and Ma’s investigation of steel frames having semi-rigid joint connections between beams and columns. The important difference lies in the assumptions made about the rotational stiffness of the joints; that is, the extent to which the joints are able to transmit moments at various levels of rotation. Fig. 1 gives typical moment–rotation curves for skeletal steel beam-column connections, from which it may be seen that even the simplest practical connection may possess significant rotational stiffness and be able to transfer moments between the connected members.

Keywords: buildings, structures & design; brickwork & masonry; composite structures

Notation

- \(B\) panel length
- \(F\) collapse load
- \(H\) panel height
- \(M_p\) plastic moment capacity
- \(K\) ratio of ultimate moment of the joint to plastic moment of the beam
- \(m_0\) non-dimensional ratio for frame/wall strength
- \(t_v\) panel thickness
- \(\alpha\) panel reduction factor
- \(\gamma_m\) material safety factor
- \(\gamma_p\) penalty factor
- \(c_c\) crushing stress
- \(c_e\) effective crushing stress
- \(\phi\) angle of rotation


\(^3\) The results of several tests carried out by Davison and extracted from the database are shown in Fig. 2. The results are

Ultimate strength levels

5. A large volume of test data is now available covering a wide variety of beam-column connections, and in 1988 a computerized databank was established in the Department of Civil and Structural Engineering at the University of Sheffield.\(^3\) The results of several tests carried out by Davison\(^4\) and extracted from the databank are shown in Fig. 2. The results are
illustrated in Table 1 which indicates that it might be convenient to use ultimate moment magnitudes of $0.18M_p$ and $0.75M_p$ for the pinned to partial strength and the partial strength to full strength boundaries, respectively, although for full strength connections it may be necessary to set the ultimate bending moment boundary higher than $0.75M_p$, perhaps even up to the full $M_p$ of the beam.

**Limit analysis of panels**

**Yield criterion for frame**

6. In this section the enclosing frames are assumed to behave in a rigid plastic manner with the yield criterion

$$|M| \leq M_p$$  \hspace{1cm} (1)

**Shear mode of infill panel**

7. The infill is assumed to have the square yield criterion as shown in Fig. 3, where the stress axes are the principal stresses. This criterion was proposed by Nilson et al., and has been used by Wood, and Sims for the analysis of infilled frames. For a rigidly jointed infilled panel a basic shear mode $S$ of collapse was proposed in which it was assumed that plastic hinges developed at the four corners of the bounding frame. This mode is also expected for an infilled frame with semi-rigid joints. The probability of the infill material behaving in an ideal, perfectly plastic manner is less than that of the surrounding frame, so a penalty factor $\gamma_p$ was introduced by Wood. The penalty factor is applied to lower the crushing stress, $\sigma_c$, of the infill material to produce an effective crushing stress $\sigma_c^*$, such that

$$\sigma_c = \gamma_p \sigma_c^*; \quad \gamma_p < 1$$  \hspace{1cm} (2)

The factor $\gamma_p$ is dependent on the relative strength of the infill material and bounding frame. An illustration of this parameter is given in Fig. 4 (taken from Wood).

**Composite shear mode of infill and frame**

8. May determined the upper bound solution for the $S$ mode of collapse, in which the frame is assumed to sway by an angle $\phi$ (Fig. 5(b)), with the formation of hinges at the four corners of the frame. For small displacements, a rectangular frame $B \times H$ will not require any expansion of the wall but only a pure shear strain $\phi$. This gives principal strains of $\pm \phi/2$ and hence, from the normality rule and the yield criterion, the wall will have associated principal stresses of 0 and $-\sigma_c$. In the $xy$ coordinate system this leads to shear stresses of $\pm \sigma_c/2$, together with an accompanying hydrostatic stress of $-\sigma_c/2$ as indicated in Fig. 5(a). Hence, equating the work done by the external loading to the energy dissipated in the frame and panel at the point of failure, the following equation is obtained:

$$FH\phi = 4kM_p\phi + \frac{1}{2}\sigma_c t_e B H$$  \hspace{1cm} (3)

or

$$F = \frac{4kM_p}{H} + \frac{1}{2}\sigma_c t_e B$$  \hspace{1cm} (4)

**Table 1. Values of the ultimate moment $k$ and $\alpha$ for various connection types**
Define $f$ as
\[
 f = \frac{F}{(4M_p/H) + \frac{1}{2} \sigma_w t_w B} \tag{5}
\]
and $m_0$ by
\[
 m_0 = \frac{8M_p}{\sigma_w t_w B^2} \tag{6}
\]
For the S mode of collapse, an upper bound is given by
\[
 f = \frac{km_0(B/H) + 1}{m_0(B/H) + 1} \tag{7}
\]
A lower bound on the collapse load was also determined and found to be identical to the upper bound, provided that the hinges form at the corners in the bounding frame.

**The design procedure**

9. Thus a practical procedure for evaluating the strength of an infilled panel is the following straightforward procedure:

(a) Compute the value of $m_0$ for the panel from equation (6).
(b) Determine $\gamma_p$ from Fig. 4.
(c) Determine the factor $k$ from Table 1.
(d) Calculate $f$ from equation (7) using $m = m_0/\gamma_p$.
(e) Evaluate the collapse load $F$ using
\[
 F = f \left( \frac{4kM_p}{H} + \frac{1}{2} \sigma_w t_w B \right) \tag{8}
\]

10. The values of $\gamma_p$ are shown in Fig. 4 plotted against the nominal values of $m_0$ using the full crushing strength $\sigma_c$. It is intended that the designer starts with nominal values of $m_0$ (equation (6)), ascertains $\gamma_p$, to re-calculate $m = m_0/\gamma_p$, then re-calculates $f$ from equation (7), and finally calculates the collapse load in equation (8) using $\gamma_p \sigma_c$. The relevant $\gamma_m$ is a material partial safety factor to allow for imperfections and to provide a reserve of strength. It is noted that BS 5628: Part 3 already provides an adequate margin of safety against the attainment of the ultimate limit state.

**Parametric study**

11. The empirical penalty factor $\gamma_p$ proposed by Wood has been criticized by some authors who did several tests with different parameters not considered by Wood. The design method used by Wood is not always conservative and can given an overestimate of the value of the design collapse load, probably due to the fact that it attempts to suggest a universal criterion for all infill panels.

12. Several parameters have been investigated by Nadjai to adjust the expressions proposed by Wood for the design of collapse panels.
load. These are:

(a) Types of semi-rigid beam–column connections:
   (i) extended end-plates,
   (ii) flush end-plates,
   (iii) flange cleats, and
   (iv) web cleats.

(b) The presence of infill in multistorey frames.

(c) The presence of strong and weak infill in stiff and flexible steel frames.

(d) The influence of variation in the elastic properties of the infill.

13. To investigate the above parameters, a computer program was developed by Nadjai using a non-linear finite-element approach, which takes into account the yielding of steel in both the members and the connections, cracking and crushing of the infill, and the non-linear interaction at the frame–infill boundary.

Program validation

14. Among the several tests used in the validation are: (a) a small-scale steel open frame using semi-rigid connection (Fig. 6) tested by Stelmack and (b) a full-scale infilled steel frame rigidly jointed and pin connected (Fig. 7) tested by Dawe and Seah.

15. Stelmack's test frame, which consisted of a two-storey frames with flexible connections, was analysed using the present computer program. The joint connections were idealized with a trilinear representation of the real $M - \phi$ response. Fig. 8 shows the lateral load–displacement curves for the first and second storey, along with the predicted results. In general, the analytical load deflection curves agree reasonably well with the test results.

16. Dawe and Seah's tests, in which the infilled frame was constructed and tested with rigid and pinned joint connections between the steel members. It is apparent from Fig. 9 that the use of a fully articulated frame results in a reduction in the ultimate load of over 50%. The major crack load is also reduced by about 25%. The analytical load–deflection curves shown in Fig. 9 agree reasonably well with Dawe and Seah's test results. Although no test information is yet available for frames with semi-rigid joints and infill panels, it can be expected that the results for these frames will fall somewhere between those for the frames with rigid and pinned joints.

Analytical investigation

17. A two-storey infilled steel frame is presented to show that the presence of semi-rigid joints has an important influence on the behaviour of infilled frames. The steel frame used had material properties consistent with those of the frame tested by Davison. The columns were 152 × 152 UC23 sections and the beams were 254 × 102 UB22 sections. A value of 210 kN/mm$^2$ was assumed for the modulus of elasticity. The infill material was assumed to be uniform and to have mechanical properties corresponding to those of blockwork made of structural 100 mm thick solid blocks with 10 N/mm$^2$ nominal strength using the material properties given by Riddington. The values of the shear stiffness and the coefficient of friction were taken from the shear box tests reported by King and Pandey.

18. From the load–deformation curves shown in Fig. 10 it is seen that, in the elastic range, much of difference in the lateral deflection occurs due to the use of different types of connection; the beam–column joints have a significant influence on the behaviour of the steel structures as they directly control the lateral stiffness, the ultimate load of the structure. The load–drift curves of the frames (Fig. 10) illustrate the decrease in load-carrying capacity of the frame with increasing connection flexibility.

19. Not surprisingly, it was found during the analysis that the infilled steel frame structures...
were capable of withstanding larger ultimate loads. The load–deflection curves (Fig. 11) show that in the initial stages of loading, when response is in the elastic range, there is not much difference between frames with different connections because the frame soon comes into contact with the infill panel, which stiffens the structure substantially. After a certain level of loading, the effect of joint response starts to become more significant. The load–drift curves of the frames (Fig. 11) illustrate that the load-carrying capacity of the infilled frames increases with higher connection rigidity.

20. It is tentatively suggested, based on a limited parametric study and experimental tests, that Wood’s penalty factor might be relaxed. Experimental tests and analytical examples are used to demonstrate the predicted design collapse load using the hand method, and using a penalty factor. The new formula for the collapse load is

\[ F = f \left( \frac{4kM_p}{H^2} + \frac{1}{2} \frac{\gamma_0 \sigma}{\gamma_m} t_m B \right) \]  

(9)

where \( \alpha \) is a panel reduction factor (discussed in the next section).

21. The use of a reduction factor \( \alpha \) is reasonable because the behaviour of masonry in a frame is different from that which occurs in tests to determine its characteristic strength. The flow chart shown in Fig. 12 differentiates between ideal analyses and a practical design process, which needs to incorporate factors of safety to cater for practical variability in the form of values for \( \gamma_m \). The factor \( \alpha \) is then derived from a comparison of predicted and experimental test results. Table 1 gives indicative values derived from such computations using a very limited set of experimental values.

**Design examples**

**Example 1**

22. The method is demonstrated using the frame shown in Fig. 7, which was tested by Dawe and Seah.\(^{15}\) Two types of joint were considered: (a) flexible approximating to pinned, and (b) rigid. The corresponding experimentally observed ultimate capacity shear loads for the two frame–joint types were 267 and 556 kN, respectively. The infill panel was 3600 mm long and 2800 mm high. The panel consisted of 400 mm × 200 mm × 200 mm concrete blocks placed in running bond within a surrounding steel frame fabricated using W10 × 39 columns and a W8 × 31 beam. The plastic moment capacity of the beam was \( M_p = 137 \text{ kN} \cdot \text{m} \). The characteristic strength of the blockwork \( \sigma \) was 8 N/ mm\(^2\) and the partial safety factor \( \gamma_m \) was taken as 1·0 for the calculation.

23. Remember that, in practice, \( \gamma_m \) would be
about 2.5, to cater for the inherent variability of the blockwork material.

24. **Flexible joints without reduced penalty factor.**

(a) Equation (6) gives

\[ m_0 = \frac{8 \times 137 \times 10^6}{8 \times 200 \times 3600} = 0.05 \]

(b) From Fig. 4, \( \gamma_0 = 0.32 \) and \( m = 0.05/0.32 = 0.16 \).

(c) For a flexible joint, from Table 1 \( k = 0.18 \).

(d) Equation (7) gives

\[ f = \frac{0.18 \times 0.16 \times (3.6/2.8) + 1}{0.16(3.6/2.8) + 1} = 0.86 \quad (10) \]

(e) The collapse load is, therefore,

\[ F = 0.86 \left[ \frac{4 \times 0.18 \times 137}{2.8} + \left( \frac{1}{2} \times 0.32 \times 8 \right) \times 3.6 \times 200 \right] \]

\[ = 0.86(35\text{frame}) + 922(\text{wall}) \]

\[ = 952\text{kN} \]

This compares with a test value of 267kN.

25. **Note:** The test results show that the formula gives an overestimate of the capacity of the wall if safety factors of 1 are used. The steel resistance is reliably known; therefore, any correction should be made to the panel strength and its interaction with steel.

26. The panel resistance is calculated as \( 0.86 \times 922 = 793\text{kN} \). The panel resistance expected is \( 267 - (0.86 \times 35) = 237\text{kN} \). The reduced factor \( \alpha \) calculated for pinned connections is \( \alpha = 793/237 = 3.3 \).

27. **Flexible joints with reduced penalty factor.**

Using the reduced factor \( \alpha = 3.3 \) equation (9), the collapse load is

\[ F = 0.86 \left[ \frac{4 \times 0.18 \times 137}{2.8} + \left( \frac{1}{2} \times 0.32 \times 8 \right) \times 3.6 \times 200 \right] \]

\[ = 0.86(35\text{frame}) + 280(\text{wall}) \]

\[ = 271\text{kN} \]

This compares with a test value of 267kN.

28. **Rigid joints.** Using a partial safety factor of 1 for the infill (\( \gamma_{\text{inf}} = 1.0 \))

(a) As above.

(b) As above.

(c) For a rigid joint, from Table 1 \( k = 0.75 \).

(d) Equation (7) gives

\[ f = \frac{0.75 \times 0.16 \times (3.6/2.8) + 1}{0.16(3.6/2.8) + 1} = 0.95 \quad (11) \]

(e) The collapse load is, therefore,

\[ F = 0.95 \left[ \frac{4 \times 0.75 \times 137}{2.8} + \left( \frac{1}{2} \times 0.32 \times 8 \right) \times 3.6 \times 200 \right] \]

\[ = 0.95(147\text{frame}) + 922(\text{wall}) \]

\[ = 1016\text{kN} \]

This compares with a test value of 556kN.

29. The panel resistance is calculated as \( 0.95 \times 922 = 876\text{kN} \). The panel resistance expected is \( 556 - (0.95 \times 147) = 416\text{kN} \). The reduced factor \( \alpha \) calculated with rigid connections is \( \alpha = 876/416 = 2.1 \).

30. **Rigid joints with reduced penalty factor.** Using the reduced factor \( \alpha = 2.1 \) and equation

\[ F = 2.1 \left[ \frac{4 \times 0.95 \times 137}{2.8} + \left( \frac{1}{2} \times 0.32 \times 8 \right) \times 3.6 \times 200 \right] \]

\[ = 2.1(147\text{frame}) + 922(\text{wall}) \]

\[ = 1370\text{kN} \]

This compares with a test value of 556kN.
(9), the collapse load is
\[ F = 0.95 \left[ 4 \times 0.75 \times 137 \div 2.8 + \left( \frac{1}{2} \times 0.32 \div 21 \times 8 \right) \times 3.6 \times 200 \right] \]
\[ = 0.95(147\text{ frame}) + 439\text{ (wall)} \]
\[ = 557\text{ kN} \]
This compares better with the last value of 556 kN, then the value found by Wood’s method needs a reduction factor of over 2 to be more practical.

**Example 2**
31. An analytical example was chosen from the parametric study^{13} of one single infilled steel frame with different joint connections (Fig. 13). The connections selected are rigid, extended end-plate, flush end-plate, flange cleats, and web cleats with ultimate loadings of 197, 174, 150, 121 and 107 kN, respectively (Fig. 14). A comparison of the results obtained with design method and with the analysis is given in Table 2.

32. It can be seen from Table 2 that the hand design method gives a gross overestimate of the capacity of the wall if a penalty factor \( \gamma_p \) of 0.37 is used with no further reduction factor. Although the connections constitute only a small portion of the steel frame, their effect is significant in the overall structural performance (Table 2). For example, the ultimate resistance of the infilled frame is reduced by about 46\% due to a change from fully rigid to web cleat joints.

33. Bearing in mind the need for simple values for the reduction factor \( \alpha \), an analysis of the results has led the following tentative suggestions for suitable values:

- Rigid joint connections \( \alpha = 2.0 \)
- Extended end plate \( \alpha = 2.2 \)
- Flush end plate \( \alpha = 2.5 \)
- Flange cleats \( \alpha = 2.8 \)
- Web cleats \( \alpha = 3.2 \)

More accurate values may well be related, not specifically to joint type, but to the ratio of joint strength and stiffness to panel strength and stiffness. However, there is a practical need to keep the method as simple as possible if it is to be developed into an approach accepted by practitioners.

**Conclusions**
34. The test and analysis results show that Wood’s method for pin jointed infilled frames without a penalty factor can give an overestimate of the capacity of a wall. The use of the penalty factor suggested by Wood can be conservative if used for frames with the full spectrum of joints, as the characteristics of joints between beams and columns is very influential on the response of such a structure.

35. The main aim of this investigation was to suggest a design method for infilled steel frames having any type of beam–column connection. A simple system for the classification of steel beam–column connections in terms of stiffness and ultimate strength has been presented that is practical for use by designers and fabricators of structural steel building structures because it does not require the use of detailed connection performance data.

36. It has been demonstrated that the type of joint connection has an important influence on the capacity of the composite structure. The results presented show that there are substantial benefits to be gained from including the effects of beam–column connections in the calculation of the strength of these structural frames.
Table 2. Ultimate load values obtained with the design method and with finite-element analysis

<table>
<thead>
<tr>
<th>Joint type</th>
<th>Joint factor, k</th>
<th>Load factor, f</th>
<th>Penalty factor, γp</th>
<th>Reduction factor, α</th>
<th>Design failure load, F: kN</th>
<th>Calculated failure load, F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rigid</td>
<td>1.0</td>
<td>1.0</td>
<td>0.37</td>
<td>2.0</td>
<td>679</td>
<td>197</td>
</tr>
<tr>
<td>Extended end-plate</td>
<td>0.75</td>
<td>0.98</td>
<td>0.37</td>
<td>2.2</td>
<td>652</td>
<td>177</td>
</tr>
<tr>
<td>Flush end-plate</td>
<td>0.86</td>
<td>0.95</td>
<td>0.37</td>
<td>2.5</td>
<td>611</td>
<td>156</td>
</tr>
<tr>
<td>Flange cleats</td>
<td>0.28</td>
<td>0.94</td>
<td>0.37</td>
<td>2.8</td>
<td>600</td>
<td>135</td>
</tr>
<tr>
<td>Web cleats</td>
<td>0.18</td>
<td>0.93</td>
<td>0.37</td>
<td>3.2</td>
<td>588</td>
<td>112</td>
</tr>
</tbody>
</table>

BS 5628: Part 3: 1978
γm = 3.1 special
γm = 2.8–3.0 normal

Fig. 12. Flow chart of the design procedure
References