FE-EFGM COUPLING USING MAXIMUM ENTROPY SHAPE FUNCTIONS AND ITS APPLICATION TO SMALL AND FINITE DEFORMATION

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ABSTRACT

The element-free Galerkin method (EFGM) is superior to its counterpart the finite element method (FEM) in terms of accuracy and convergence but is computationally expensive. Therefore it is more practical to use the EFGM only in a region, which is difficult to model using the FEM, while the FEM can be used in the remaining part of the problem domain. In the conventional EFGM, moving least squares (MLS) shape functions are used for the approximation of the field variables. These shape functions do not possess the Kronecker delta property and it is therefore not straightforward to couple the EFGM with the FEM. Local maximum entropy (max-ent) shape functions are an alternative way to couple the EFGM and FEM. These shape functions possess a weak Kronecker delta property at boundaries, which provides a natural way to couple the EFGM with the FEM as compared to the MLS basis functions, which need extra care to properly couple the two regions. This formulation removes the need for interface elements between the FEM and the EFGM, unlike the approach adopted by most researchers. This approach is verified using benchmark problems from small and finite deformation.

1 INTRODUCTION

The EFGM [1] is an excellent tool for the solution of boundary value problems in computational mechanics. In the conventional EFGM, MLS shape functions are used for the approximation of the field variables, background cells are used for numerical integration and essential boundary conditions are implemented with Lagrange multipliers. Compared to the FEM, the EFGM is more accurate with a high rate of convergence, post processing is simple and there is no need for element connectivity information, so nodes can be added and deleted without computationally expensive remeshing. These advantages make the EFGM very suitable for certain classes of problems, e.g., those with large deformation and moving boundaries.

On the other hand due to the use of the MLS basis functions, the EFGM is computationally more expensive, especially for three-dimensional and nonlinear problems. At each integration point, a system of linear equations are solved to calculate the MLS shape functions, a high integration order needs to be used for accurate results due to the non-polynomial nature of the shape functions and essential boundary conditions cannot be implemented directly as in the case of FEM due to the lack of Kronecker delta property. Therefore it is more practical to use the EFGM only in a region where high accuracy is required, while FEM can be used in the remaining part of the problem domain. Coupling between EFGM and FEM is therefore a key issue.

The coupling between EFGM and FEM was first proposed in [2] based on interface elements. Hybrid shape functions for these elements, consisting of both the FEM and the EFGM shape functions, are formed using ramp functions. A detailed overview and analysis of the coupling between meshless and
FEM can be found in [3]. In this paper a new way of coupling the EFGM and the FEM using local max-ent shape functions is proposed. These shape functions possess a weak Kronecker delta property at the boundaries. This property provides a natural way to couple the EFGM with the FEM as compared to the MLS basis functions, which need extra care properly to couple the same regions.

2 LOCAL MAXIMUM ENTROPY SHAPE FUNCTIONS

For completeness a very brief overview of local max-ent shape functions are given, the detail can be found in [4] and the references therein. The two-dimensional formulation is given here, which is straightforward to modify for one- or three-dimensions. The expression for the local max-ent shape function \( \phi \) for node \( i \) at point of interest \( x = [ x \ y ]^T \) is written as

\[
\phi_i = \frac{Z_i}{Z}, \quad Z_i = w_i e^{-\lambda_1 \tilde{x}_i - \lambda_2 \tilde{y}_i}, \quad Z = \sum_{j=1}^{n} Z_j.
\] (1)

Here \( w_i \) is the weight function associated with node \( i \), evaluated at point \( x \), \( \tilde{x}_i = x_i - x \) and \( \tilde{y}_i = y_i - y \) are shifted coordinates, \( n \) is the number of nodes in support of point \( x \) and \( \lambda_1 \) and \( \lambda_2 \) are Lagrange multipliers and can be written as

\[
\lambda = \text{argmin } F(\lambda), \quad \text{where } F(\lambda) = \log(Z),
\] (2)

i.e. the set of values for which \( F \) attains its minimum. The expressions for the shape function derivatives are

\[
\nabla \phi_i = \phi_i \left( \nabla f_i - \sum_{i=1}^{n} \phi_i \nabla f_i \right),
\] (3)

where

\[
\nabla f_i = \frac{\nabla w_i}{w_i} + \lambda + \tilde{x}_i \left[ H^{-1} - H^{-1} A \right], \quad A = \sum_{k=1}^{n} \phi_k \tilde{x}_k \otimes \frac{\nabla w_k}{w_k}.
\] (4)

3 NUMERICAL EXAMPLES

One- and two-dimensional numerical examples from small and finite deformation are now given to demonstrate the implementation and performance of the current approach in linear and nonlinear problems.

3.1 Small deformation

In this section the behaviour of a one-dimensional bar of unit length subjected to body force and fixed at a point \( x = 0 \) and a two-dimensional cantilever beam subjected to parabolic traction at the free end are examined. In the 1D example the modulus of elasticity \( E = 1 \). Figure 1(a) shows the FEM and the EFGM regions with the shape functions for all the nodes in the problem domain and Figure 1(b) shows solution for the nodal displacement. The two-dimensional example is the famous Timoshenko beam problem, for which the analytical solution is given in [5]. The problem is solved for the plane stress case with \( P = 1000, \nu = 0.3, E = 30 \times 10^6 \), all in compatible units. Figure 2(a) shows the FEM and EFGM regions, Figure 2(b) shows the deflection at \( y = 0 \) and Figures 2(c) and 2(d) shows the normal and shear stresses \( \sigma_{xx} \) and \( \sigma_{xy} \) at \( x = L/2 \).

3.2 Finite deformation

An updated Lagrangian formulation is used to model problems subjected to finite deformation. Here the deformation gradient \( F \) provides the link between the current and the reference configurations. The logarithmic strain \( \varepsilon = \ln(v) \) and Kirchhoff stress \( \tau = J\sigma \) are used in this case, where \( v = \sqrt{FF^T} \) is the left stretch matrix, \( J \) is the determinant of the deformation gradient and \( \sigma \) is the Cauchy or true stress. Two problems are solved in this case: a uniform loaded rectangular plate with simply supported edges, the analytical solution for which is available in [6] and a cantilever beam with uniform loading, the reference solution for which is given in [7]. The geometry for the plate problem is shown in Figure 3(a), due to symmetry only half of the plate is modelled. The problem is solved with \( q = 40, h = 0.2, \)
\[ L = 10, \quad E = 10^{7}, \quad \nu = 0.25 \text{ all in compatible units with 20 load steps. Half of the plate is modelled with the EFGM and other half is modelled with the FEM. Figure 3(b) shows the pressure vs \( w/h \) for the plate problem. The beam problem is solved with \( q = 10, \quad h = 1, \quad L = 10, \quad E = 1.2 \times 10^{4}, \quad \nu = 0.2 \) with 20 load steps all in compatible units. Half of the beam is modelled with the EFGM and other half is modelled with the FEM. Figure 4(a) shows the deformed and un-deformed configuration at the end of solution with the EFGM and the FEM regions for the problem. Figure 4(b) shows the pressure vs \( u_y/L \) for the beam problem.}

4 Conclusion

In this paper a new method of coupling the FEM with the EFGM based on local max-ent shape functions is proposed. Numerical examples are solved from small and finite deformation. In the small deformation case, numerical results for displacement and stresses are compared with the available analytical
solution, which shows excellent agreement. In finite deformation case, the analytical solution for the pressure vs deflection curve is available for the plate problem, while for the beam problem a reference solution is used for comparison. In both cases the numerical results are in a very good agreement with the analytical/reference solutions.

References


