

Accounting for Locational, Temporal, and Physical Similarity of Residential Sales in Mass Appraisal Modeling: Introducing the Development and Application of Geographically, Temporally, and Characteristically Weighted Regression (GTCWR)

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Abstract

Geographically weighted regression (GWR) has been recognized in the assessment community as a viable automated valuation model (AVM) to help overcome, at least in part, modeling hurdles associated with location, such as spatial heterogeneity and spatial autocorrelation of error terms. Although previous researchers have adjusted the GWR weights matrix to also weight by time of sale or by structural similarity of properties in AVMs, the research described

in this paper is the first that has done so by all three dimensions (i.e., location, structural similarity, and time of sale) simultaneously. Using 24 years of single-family residential sales in Fairfax, Virginia, we created a new locally weighted regression (LWR) AVM called geographically, temporally, and characteristically weighted regression (GTCWR) and compared it with GWR-based models with fewer weighting dimensions.

GTCWR was the only model to achieve IAAO-accepted levels of the coefficient of dispersion (COD), price-related differential (PRD), price-related bias (PRB), and median assessment-to-sale price ratio in both the training and testing samples, although it did not fully correct the existence of heteroscedasticity. With lower PRD and PRB levels, the application of temporal weighting to this data set did appear to help reduce indicators of vertical inequity. Along with an equitable, uniform, and defensible methodology that mirrors the sales

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comparison, GTCWR presents a new AVM that demonstrates an ability to value over 24 years of sales at IAAO standard levels, without the creation and implementation of time-based variables, the trimming of outliers, and time-intensive model specification and calibration.

Introduction

Inequitable real estate valuations used for property tax assessments lead to excessive burdens of time and cost for taxpayers and local governments alike. For municipalities, such inconsistencies result in a higher number of tax appeals. More equitable, research-backed valuations help local governments defend and explain their assessments, ideally (for local governments) resulting in fewer appeals, more appeals settled out of court, and more court cases won. Improved valuations help shift funds more equitably by helping to correct for previously undervalued and/or overvalued properties. By increasing the precision of valuation models, this research aimed to mitigate the financial costs and political ramifications of subpar valuations.

The sales comparison approach is a real estate valuation method by which an appraiser arrives at an estimate of value for a property (as of a specified valuation date), based on sales that have occurred within a specific time frame (generally, but not exclusively, 6–12 months prior) and that are both structurally and geographically similar to the subject property (Fannie Mae 2017). If executed properly, this approach produces a reasonable and defensible estimate of value based on what a given property would sell for in the specified market. This approach identifies determinants of value based on supply and demand (Gloude-mans and Almy 2011).

Although the comparable sales approach arguably has its advantages (e.g., fewer computational requirements than other approaches, familiarity among the real estate community, logical and economical explanation and defensibility), it is potentially prone to errors. Small sample sizes, subjectivity of comparable selection, user error, and failure to exclude non-arm's-length transactions are examples of potential reasons the comparable sales approach may fail to produce consistent, uniform, and equitable results. Such errors are greatly mitigated with the implementation of an AVM.

AVMs are programmed statistical models used to predict values of large sets of real estate properties at once. These models (often referred to as computer-assisted mass appraisal [CAMA] models or hedonic pricing models) have been steadily increasing in popularity in both the public and private sectors since the 1970s, primarily because of advancing technology, increasing computational power and speed,

and changing methodologies that continuously make valuations more accurate and easier to execute. Although there are a number of types of models and algorithms that fall under the AVM umbrella, the most common types are based on ordinary least squares (OLS) multiple regression. When combined with professional judgment, AVMs been shown to improve valuation equity, uniformity, and accuracy. Modifications that allow AVMs to emulate the comparable sales approach have resulted in better performances.

Geographically Weighted Regression

LWR is a modeling technique that has been shown to significantly improve the predictability power of traditional OLS-based AVMs (Brunsdon, Fotheringham, and Charlton 1996; McMillen 1996; Brunsdon 1998). Arguably, the most prevalent LWR methodology that has been implemented as an AVM is GWR. Similarly to the comparable sales approach, GWR allows observations in closer geographic proximity to the subject sale to receive more consideration than those further away (Fotheringham, Brunsdon, and Charlton 2002). Because real estate markets behave so differently over geographic space and location plays such a large role in price formation, conventional OLS models are often unable to accurately account for spatial variation. This results in a correlation of error terms across a geographic plane (spatial autocorrelation). Although spatial consideration in the form of dummy variables or distance coefficients can help improve models, it may fail to fully correct for spatial autocorrelation, and hedonic parameter averages may be skewed or averaged out (Fotheringham, Brunsdon, and Charlton 2002; McMillen and Redfean 2010).

IAAO develops and maintains statistical standards by which assessing jurisdictions can measure, track, and compare valuations with respect to various performance measures, including assessment uniformity and equity (IAAO 2003). Valuation estimates produced by GWR AVMs have been shown to achieve superior results with respect to such IAAO standards when compared to valuations produced by OLS AVMs (Borst and McCluskey 2008; Moore 2009; Moore and Myers 2010; Lockwood and Rossini 2011; McCluskey et al. 2013; Bidanset and Lombard 2014a).

Modifying GWR AVMs to Allow for Additional Weighting Parameters

GWR AVMs that allow the weighting of additional parameters have demonstrated a higher predictability power than those that do not, both inside and outside of the property tax industry. The introduction of a temporal weighted

regression (TWR) modifies the weighting scheme with a time component, in which observations that occurred more recently to the regression point receive a higher weight than those that occurred further away. This spatio-temporal LWR approach is often referred to as GTWR, and it is suggested that it outperforms both GWR and OLS with respect to housing price predictions (Crespo, Fotheringham, and Charlton 2007; Huang, Wu, and Berry 2010; Fotheringham, Crespo, and Yao 2015). Borst (2014) demonstrated the ability of spatiotemporal weighting to improve assessment valuations, as evidenced by more uniform and equitable results across stratum.

In 2006, Shi, Zhang, and Liu modified the spatial weights matrix of a GWR model (used to predict spatial patterns of trees) to also take into account similarity of tree attributes, so that not only trees closer to the subject tree but also those trees more physically similar to the subject tree (e.g., trunk diameter) are more heavily considered. This resulted in smaller residuals and improved predictions. Similarly, Moore and Meyers (2010) utilized such a spatial-attribute weighting function (coined geographic-attribute weighted regression [GAWR]) that achieves higher accuracy and uniformity than an AVM with a spatial weighting function (e.g., GWR) alone.

Justification for Research

Because independent weighting of time and attribute improves GWR AVMs, a logical hypothesis follows that weighting by all three dimensions simultaneously would produce optimal property valuations. Surprisingly, there has yet to be research that has done so (to our knowledge). Jiang et al. (2013) modified the GTWR weights matrix of Huang, Wu, and Barry (2010) to allow for various attribute similarities of sales, with respect to both physical characteristics and location, and disseminated their research at the 12th International Conference on GeoComputation at Wuhan University.

They did not, however, specify whether models were applied to and validated using holdout samples. This is important as decreasing bandwidths in GWR models can result in overfitting (Guo et al. 2008), and applying additional weighting components will therefore likely have a similar effect, as both reduce the magnitude other observations have in fitting each regression point. This is also particularly relevant in the assessment community, as the IAAO's AVM standard states: "the AVM also should be tested on a holdout sample, which is a set of properties and their selling prices that were not used in the calibration process" (IAAO 2003).

The research described in this paper addressed the current literature gap by creating and evalu-

ating a model that is an extension of GWR and that accounts for variations in locational, temporal, and physical similarity of properties by incorporating concurrent spatial, temporal, and physical weighting functions and by evaluating the respective impacts of their modifications on uniformity and equitability of CAMA valuations used for property tax purposes. This was done by replacing the GWR weighting function with a locally weighted regression scheme that simultaneously:

- Weighted observations (from 0 to 1) based on geographic proximity to the subject property, in which the weight decays as the observation becomes further away.
- Weighted observations (from 0 to 1) based on time proximity to the subject property, in which the weight decays as the time since the sale increases.
- Weighted observations (from 0 to 1) based on some calculated degree of similarity in characteristics to the subject property, in which the weight decays as dissimilarity increases.

All results were validated using a randomly selected holdout sample, which was omitted from model specification and calibration.

Although the primary intended audience of this paper is professionals in property tax assessment valuation, this new AVM we have coined GTCWR has implications for anyone who must produce and/or rely upon accurate, large-scale real estate valuations at a reasonable cost—portfolio valuation analysts, loan originators, mortgage companies, and other real estate professionals.

Methodology

GWR is represented by the following formula (Fotheringham, Brunson, and Charlton 2002, 61):

$$(1) y_i = \beta_0(x_i, y_i) + \sum \beta_k(x_i, y_i) x_{ik} + \varepsilon_i$$

where

y_i = i -th sale

β_0 = model intercept

β_k = k -th coefficient

x_{ik} = k -th variable for the i -th sale

ε_i = error term of the i -th sale

(x_i, y_i) = x - y coordinates of the i -th regression point.

The bandwidth in GWR specifies the radius of the weighting function. It is either fixed, based on absolute distance, or adaptive (fluctuating), based on a predetermined number of nearest neighbors. An optimized bandwidth may be identified based on various conditions, but most commonly it is that which corresponds to minimized cross-validation or Akaike information criterion-corrected (AICc) scores (Fothering-

ham, Brunson, and Charlton 2002) or, in the case of assessment AVMs, that which yields superior ratio study scores. For this research, an adaptive geographical bandwidth of the 33 nearest neighbors was identified as optimal.

The kernel specifies how weights are calculated and assigned to the observations. Although there are a number of kernels that may be implemented and each may have a different impact on ratio study performance (Bidanset and Lombard 2014b, 2017), an exponential weighting kernel function (equation 2) was used in this research. In GWR, an $n \times n$ spatial weights matrix is constructed to indicate the weight each observation is assigned relative to the subject, based on geographic distance.

$$(2) w_{ij} = \exp[-(|d_{ij}|/b)]$$

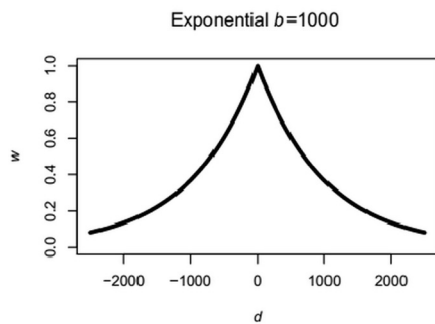
where

w_{ij} = weight applied to the j -th property at regression point i

d_{ij} = geographical distance in kilometers between regression point i and property j

b = geographical bandwidth.

A visual depiction of the respective exponential kernel weighting distribution is shown in Figure 1.



Source: Gollini et al. 2013, p. 6 (used with permission).

Figure 1: Exponential kernel weighting distribution

To exemplify how the weights matrices were constructed for this research, the existing weights matrix used in GWR was adjusted to consider, in part, both structural and temporal (time-of-sale) similarity.

To modify an existing $n \times n$ spatial weights matrix to also account for similarity of property characteristics, a separate weights matrix is calculated using each variable by which the model is to be weighted. The following kernel was used:

$$(3) w_{ij} = \exp[-|l - (j_k \div i_k)|]$$

where

w_{ij} = weight applied to the j -th property at regression point i

j = value of quantitative physical characteristic variable k of property j

i = value of quantitative physical characteristic variable k of regression point i .

A separate matrix is created for each of the following characteristics: number of rooms, number of bedrooms, and total land area. These matrices are multiplied together to produce one product matrix of characteristic similarity weights. The product matrix is multiplied by the existing spatial weights matrix already calibrated for the GWR, resulting in a spatial-attribute-weighted model we refer to as geographically and characteristically weighted regression (GCWR).

Moore and Myers (2010) similarly utilized total living area, grade, and land value and referred to this method as GAWR. Allowing for observations more similar to the subject to receive a higher consideration in price estimation than less similar properties results in a more flexible modeling approach that mirrors the comparable sales approach, as well as the actual real estate buying process—supply and demand of one submarket may not have an impact on another, or at least arguably not to the same extent as properties within the same submarket. For example, although the price of a larger house with more bedrooms and a higher land price may follow the same upward and downward cyclical market trends as a smaller house with fewer bedrooms, the two are likely not viewed by a potential buyer as substitutes.

To modify an existing spatial weights matrix to also take into consideration temporal similarity of sales, a separate weights matrix was calculated using the sale date:

$$(4) w_{ij} = \exp[-(|d_{ij}^2 \div b^2|)]$$

where

w_{ij} = weight applied to the j -th property at regression point i

d_{ij} = the temporal distance in days between regression point i and property j

b = temporal bandwidth.

This matrix is multiplied by the existing spatial weights matrix, a modification that allows for temporal consideration: GTWR (Huang et al. 2013; Fotheringham et al. 2015). The temporal bandwidth, similar to the geographic bandwidth, may be optimized with respect to a number of factors such as AICc, cross-validation, or ratio study standards. For this research, a temporal bandwidth of 1,400 days was identified as

optimal. (Optimal bandwidths likely increase [decrease] with data spanning longer [shorter] lengths of time.)

When the spatial weights matrix of GWR is multiplied by both the product matrix of characteristic similarity weights and the matrix of temporal weights, a new $n \times n$ product matrix is created, and when it is applied to the model, a locally weighted regression model is produced that accounts, simultaneously, for geographic, characteristic, and temporal similarity: GTCWR. Assigning a higher weight to sales that have occurred within a more recent window allows for local parameter estimates to be more reflective of the market at the desired time of valuation. Similarly to the comparable sales approach, sales occurring closer to the valuation date should theoretically produce more accurate value estimates than those occurring much earlier or later, also resulting in a more defensible approach.

Evaluating Results

IAAO has established statistical standards by which assessments may be evaluated for accuracy, equity, and uniformity.

The COD measures the uniformity of an assessment stratum. It is the average percentage of dispersion around the median assessment-to-sale price ratio and is calculated as follows:

$$(5) \quad COD = \frac{100}{R_m} \left[\sum_1^N \frac{|R_i - R_m|}{n} \right]$$

where

R_m = median assessment-to-sale price ratio

R_i = observed assessment-to-sale price ratio of the i -th sale

n = number of properties sampled.

IAAO ratio study standards state the COD for non-new, single-family homes should be less than or equal to 15.0. Values below 5.0 indicate potential sampling error or sales-chasing.

The PRD is an indicator of potential equity or inequity. It is represented by the following formula:

$$(6) \quad PRD = \frac{\sum_i \left[\frac{\hat{Y}_i}{Y_i} \right]}{\sum_i \left[Y_i \times \left(\frac{\hat{Y}_i}{Y_i} \right) \right] \div \sum_i Y_i}$$

where

\hat{Y}_i = predicted sale price of the i -th sale

Y_i = observed sale price of the i -th sale

n = number of properties sampled.

An acceptable PRD value falls between 0.98 and 1.03; anything below (above) this range suggests evidence of progressivity (regressiv-

ity) (Gloude-mans and Almy 2011).

The PRB is another indicator of potential equity or inequity across valuations. It represents an estimated percentage change in assessment-to-sale price ratio as values double (or halve). It is calculated using the following formula:

(7)

$$\frac{R_i - R_m}{R_m} = \beta_0 + \beta_1 \frac{\ln \left[\left(0.5 \times \left(\frac{\hat{Y}_i}{R_m} \right) \right) \times (0.5 \times Y_i) \right]}{\ln(2)} + \varepsilon_i$$

where

R_m = median assessment-to-sale price ratio

R_i = observed assessment-to-sale price ratio of the i -th sale

β_0 = model intercept

β_1 = PRB coefficient

\hat{Y}_i = predicted sale price of the i -th sale

Y_i = observed sale price of the i -th sale

ε_i = error term of the i -th sale.

An acceptable, statistically significant PRB coefficient falls between -5.00 and 5.00 , with a positive (negative) statistically significant coefficient indicating progressivity (regressivity) (Gloude-mans and Almy 2011).

Assessment-to-sale price ratios are calculated to show over- or undervaluations, both for individual properties and on the aggregate level (i.e., median assessment-to-sale price ratio).

To exemplify the impacts of adjusting the GWR weights matrix to allow for additional weights matrices, the following models were compared with respect to COD, PRD, PRB, and median assessment-to-sale price ratio:

- GWR
- GCWR
- GTWR
- GTCWR

Note that because there are no time-related variables (e.g., time splines; monthly, quarterly, or annual time variables), the purpose of the research was to evaluate, not each model's ability to handle valuation estimates over time, but the potential performance enhancement of a model by additional weighting components.

Data

The initial data consisted of 27,101 single-family home sales (of dwellings at least 3 years of age) from Fairfax, Virginia, between January 1967 and December 1990. A randomly selected subsample of 5,420 observations (20 percent) was then randomly divided into model training and testing samples ($n = 4,878$ [90 percent] and $n = 542$ [10 percent], respectively). The holdout testing sample was used for validation and to protect against model overfitting. No observations were trimmed due to outliers.

Table 1 shows the dependent variable and in-

Table 1: Descriptions and descriptive statistics of model variables

Variable	Description	Minimum	First Quartile	Median	Third Quartile	Maximum	Mean
Dependent Variable							
<i>Sale Price</i>	Price of dwelling (in 12/1990 dollars)	\$20,800	\$98,000	\$139,500	\$190,000	\$885,000	\$161,318
Independent Variables							
<i>Land Area</i>	Size of parcel (square feet)	637	637	10,452	15,566	217,316	13,199
<i>Rooms</i>	Number of rooms	4.00	4.00	8.00	8.00	16.00	7.54
<i>Baths</i>	Number of full bathrooms	1.00	1.00	2.00	2.00	5.00	2.62
<i>Half Baths</i>	Number of half-bathrooms	0.00	0.00	1.00	1.00	3.00	0.80
<i>Age</i>	Age of dwelling (years)	3.00	3.00	10.00	16.00	77.00	12.75
<i>Age2</i>	Squared age of dwelling (years)	9.00	9.00	100.00	256.00	5,929.00	251.70
<i>Age3</i>	Cubed age of dwelling (years)	27.00	27.00	1,000.00	4,096.00	456,533.00	6,862
<i>Fire</i>	Number of fireplaces	0.00	0.00	1.00	1.00	4.00	0.94

Table 2: Ratio study by performance model

Model	Training Sample				Testing Sample (Holdout)			
	COD	PRD	PRB	Median Assessment-to-Sale Price Ratio	COD	PRD	PRB	Median Assessment-to-Sale Price Ratio
GWR	17.08%	1.05	-0.08	1.02	18.75%	1.06	-0.07	1.02
GCWR	12.77%	1.04	-0.08	1.01	16.50%	1.05	-0.10	1.01
GTWR	11.33%	1.03	-0.05	1.01	15.44%	1.04	-0.03	1.01
GTCWR	7.51%	1.02	-0.05	1.00	13.38%	1.03	-0.05	1.01

dependent variables of the models, their respective descriptions, and descriptive statistics. The dependent variable, *SalePrice*, is the sale price of the home in December 1990 dollars, transformed using the monthly consumer price index (CPI) of the U.S. Bureau of Labor Statistics. *LandArea* is the size of the parcel (measured in square feet) on which the house is built. *Rooms* is the total number of rooms of the dwelling.

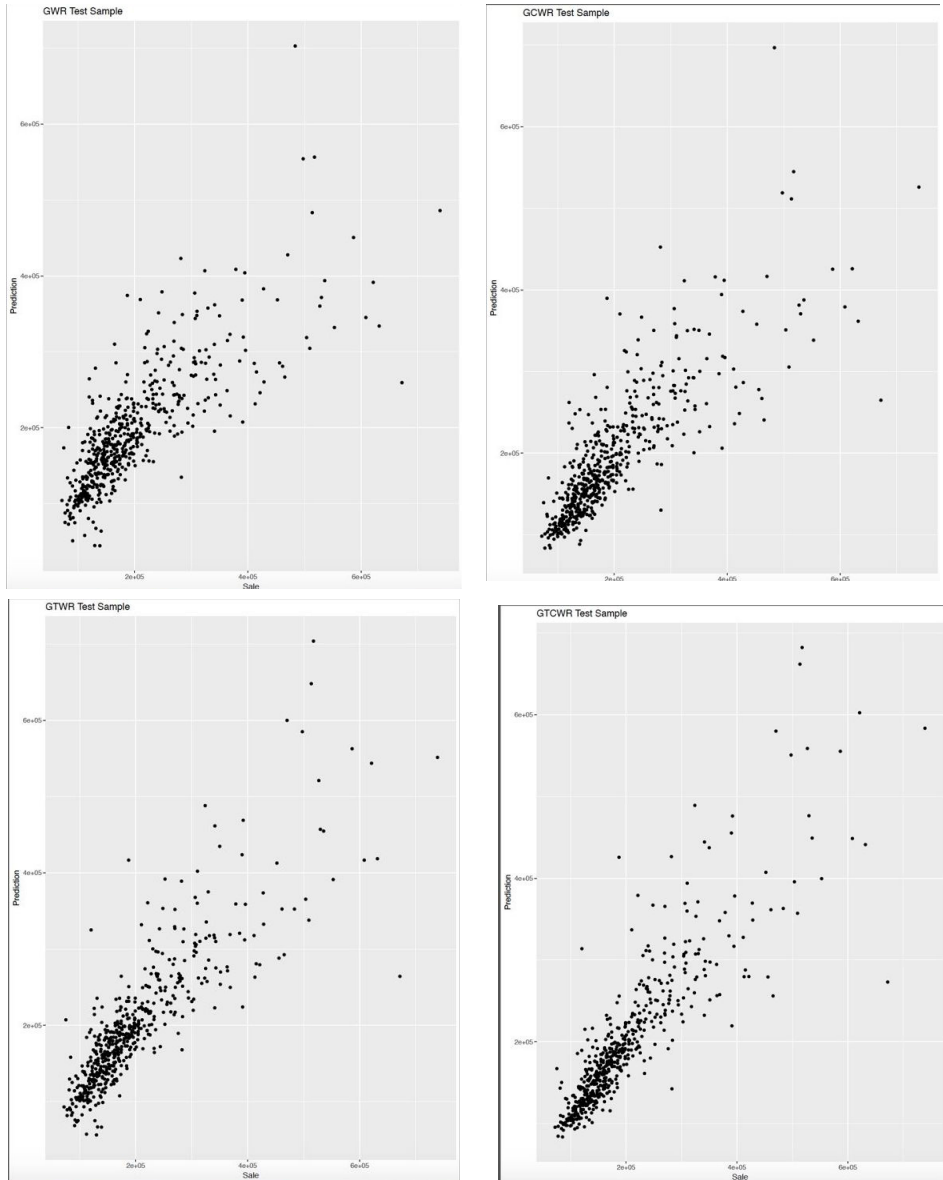
The size of a dwelling is usually represented in CAMA models by the total square feet of structure or finished living area, which is arguably both more specific and less vague than a total room count (due to variations in room size), but this information was not included in the data set. (The number of bedrooms was also provided in the data set, but did not offer an increase in explanatory power, measured by AICc).

Baths and *HalfBaths* indicate the number of full bathrooms (including a shower or bath) and half-bathrooms (sink fixture only), respectively. *Age* is the number of years since the home was built. Because the age of the dwelling demonstrated a statistically significant parabolic effect on the dependent variable, both squared (*Age2*) and cubed (*Age3*) transformations of *Age* are included. *Fire* represents the number of fireplaces within the home. All variables are reflective of the property at the time of the sale.

Results

Each model's respective assessment-to-sale price ratio based on holdout sample predictions is plotted in figure 2. Figure 2 demonstrates an increased predictability power and equity with each additional weighting component.

Figure 2: Assessment-to-sale price ratios by model (holdout testing sample)



The inclusion of each weights matrix promoted linearity across predictions, as well as a reduction in heteroscedasticity, although the largest improvement, GTCWR appears to still exhibit slight heteroscedastic errors. Each weighting component improved the baseline GWR model with respect to COD (table 2).

The most uniform predictions for both the training and testing samples (COD values of 7.51 percent and 13.38 percent, respectively) were achieved by GTCWR. GWR and GCWR failed to achieve acceptable PRD or PRB scores. Although GTWR did achieve an acceptable PRD score for the holdout sample, the

negative PRB score was not statistically significant, suggesting inconsistent indications of vertical inequity. In both the training and testing samples, GWR yielded the highest COD (17.08 percent and 18.75 percent, respectively) and the highest PRD (1.05 and 1.06, respectively). The only model that achieved IAAO-accepted levels of COD, PRD, PRB, and median assessment-to-sale price ratio across both samples was GTCWR.

In both samples, additional weights matrices reduced COD, PRD, and PRB to levels more in line with IAAO standards.

Conclusions

GWR has been recognized in the assessment community as a viable AVM to help overcome, at least in part, modeling hurdles associated with location such as spatial heterogeneity and spatial autocorrelation of error terms. By allowing model coefficients to vary over space, the mechanisms of GWR enable sales closer to a subject property to have more influence in the calculation of its respective value estimate than sales further away. This notion is similar in rationale to the sales comparison approach, in which an appraiser selects properties in the similar geographic submarket of the subject property to determine an estimate of value.

The weighting scheme of GWR inherently ignores time and structural similarity of sales used in the modeling process—two dimensions that are very much deterministic of a real estate submarket and are usually represented in a model by covariates.

Because submarkets are largely characterized by tastes, preferences, and other factors correlated with a demographic's willingness and ability to purchase a home, there is a logical and reasonable hypothesis that the prices of homes more similar to a buyer's ultimate purchase—both with respect to the characteristics and the time of the market—will have had more of an impact on its sale price than those outside of the buyer's demand. For this reason and others, the modification of GWR to allow for additional weighting considerations has become a topic of research among real estate valuation professionals.

Although previous researchers have adjusted the GWR weights matrix to also weight by time of sale or by structural similarity of properties in AVMs, there has yet to be research that has done so by all three dimensions (i.e., location, structural similarity, and time of sale) simultaneously.

Using more than 24 years of single-family residential sales in Fairfax, Virginia, we created a new LWR AVM called GTCWR and compared it with other GWR-based models with respect to IAAO ratio study performance on a hold-out testing sample. The additional LWR models evaluated were GWR (used as a baseline), GCWR, and GTWR.

Despite the fact that inflation was accounted for in the form of CPI time-adjusted sale prices, temporal weighting still had an impact on performance, indicating the presence of other time-based impacts on price. Although temporal weighting improved the models in both training and testing samples, temporal bandwidths were set at 1,400 days; models could likely benefit from the inclusion of time-based variables to help model short-run market fluctuations. Note that further research should compare the per-

formance of GTWR or GTCWR against that of GWR incorporating traditional time-based variables (e.g., time splines; reverse-month-of-sale, quarterly, or seasonal dummy variables), as well as the inclusion of such variables within temporally weighted GWR models.

GTCWR was the only model to achieve IAAO-accepted levels of COD, PRD, PRB, and median assessment-to-sale price ratio in both the training and testing samples, and the addition of temporal weighting on this data set did appear to help reduce indicators of vertical inequity, although it did not fully relieve the heteroscedasticity of the other models. It is perhaps not surprising, however, that a data set of such a long time span did not perform as well when time was not a weighting component, because there were no time variables in the baseline model.

The purpose of this research was not to compare optimal methods of accounting for heterogeneity and autocorrelation across time, space, and characteristics, but rather to highlight the fluctuation in results that can be attained simply by adjusting a weights matrix to consider multiple criteria. Still, the remarkability that 24 years of sales could attain such a performance with no time-based variables, no trimming of outliers, and minimal optimization during model specification and calibration should not be overlooked, and suggests great potential for GTCWR. Through additional research and model optimization, GTCWR performance should be expected to be further improved.

While performance statutes vary across localities, it is common for an assessor to be held to certain valuation performance levels with respect to assessment equity. Assessors should realize the potential ability of GTCWR to aid in attaining satisfactory equity levels across valuations, as well as increasing the defensibility of their values in circumstances of appeals and even court cases. There are also implications that it may benefit professionals who are elected to their position or others who may be evaluated based on the performance of valuations. As with any AVM, GTCWR may estimate overall value or modify cost manuals (depreciation schedules, factor adjustments, and so forth).

The research described in this paper has laid the foundation of GTCWR as a viable AVM for ad valorem property tax assessment purposes. There is still a great amount of research on the subject to be done, for example, identifying a method of accounting for both short-run and long-run temporal changes using an optimal balance of time variables and temporal weighting methods. Just as incorporating location-based variables into a GWR model requires careful consideration and application in order not to bias results, careful consideration will

have to be given to finding such a balance in the temporal sense as well. Various combinations of kernel specifications and bandwidth combinations should also be studied, because they have been shown to have an impact on ratio study performance (Bidanset and Lombard 2014b, 2017).

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